2D SEISMIC FACIES ANALYSIS – PROGRAM

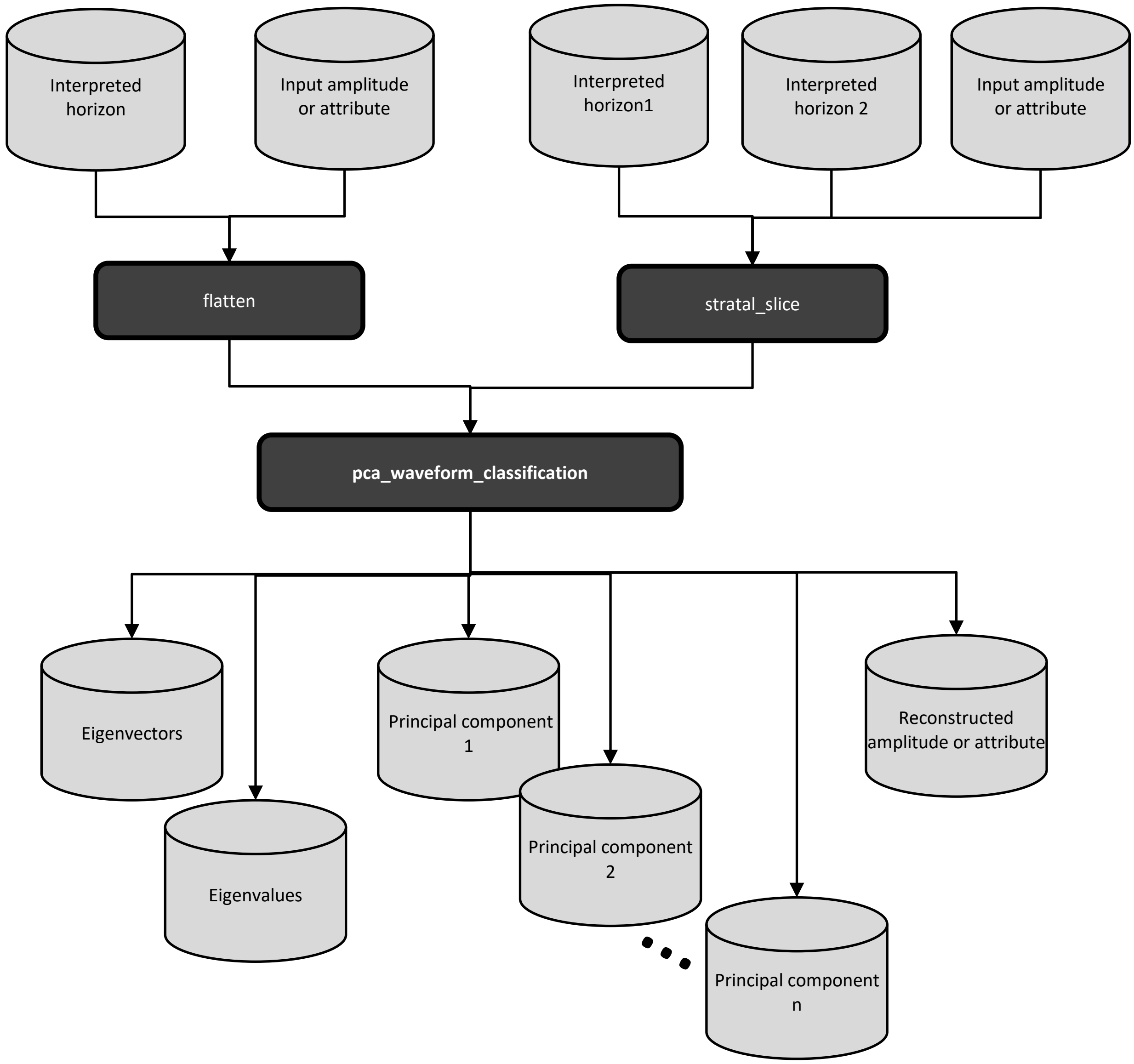
pca_waveform_classification

Contents

Computation flow chart

Principal component analysis is one of the simplest means reducing redundant high dimensional data to a much lower dimensional space. While useful, one should recognize that the meaning of principal components is mathematical rather than physical. Program **pca_waveform_classification** reads in a window of seismic amplitude, impedances, Poisson's ratio, or other attribute and outputs a suite of principal component slices. For facies analysis the input data should either be flattened or stratal-sliced along previously interpreted geologic horizons.

Formation Attributes: Program **pca_waveform_classification**



Theory: Covariance matrices, eigenvectors, eigenvalues, and principal components

The covariance matrix, **C**, is constructed by comparing each sample vector $d(t,x,y)$ to itself and all its neighbors. As an example, consider a suite of N phantom horizon slices through an M-trace seismic amplitude volume. We begin by defining the mean, μ_k for each slice:

$$\mu_n = \frac{1}{N}\sum_{m=1}^{M} d(t_n, x_m, y_m) . \quad (1)$$

The covariance matrix is then defined as

$$C_{kn} = \frac{1}{M}\sum_{m=1}^{M}\left[d(t_n, x_m, y_m) - \mu_n\right]\left[d(t_k, x_m, y_m) - \mu_k\right], \quad (2)$$

which is simply an N by N matrix of auto-correlation (if $k=n$) and cross-correlation (if $k \neq n$) coefficients of phantom horizon slices. This covariance matrix measures lateral similarities and dissimilarities (or patterns) amongst the seismic wavelets.

Any symmetric N by N matrix can be decomposed into N eigenvalue-eigenvector pairs, λ_n, and $\mathbf{v}_n$ that satisfy

$$\mathbf{C}\mathbf{v}_n = \lambda_n \mathbf{v}_n , \quad (3)$$

where n varies between 1 and N. By construction, the first eigenvector $\mathbf{v_1}$ is the vector (waveform) of unit length that best represents the energy of all vectors d_{nm}**.** If we cross-correlate this eigenvector with each trace, we obtain the first principal component:

$$p_{k1} = \sum_{n=1}^{N} d(t_n, x_m, y_m) v_{n1} . \quad (4)$$

Subtracting the data represented by the first principal component gives a residual

$$\tilde{d}_{nm} = d_{nm} - p_{n1} . \quad (5)$$

Physically, the second eigenvector $\mathbf{v_2}$ is the vector (waveform) of unit length that best represents the energy of all the residual vectors**.** The corresponding principal component is

$$p_{k2} = \sum_{n=1}^{N} d(t_n, x_m, y_m) v_{n2} , \quad (6)$$

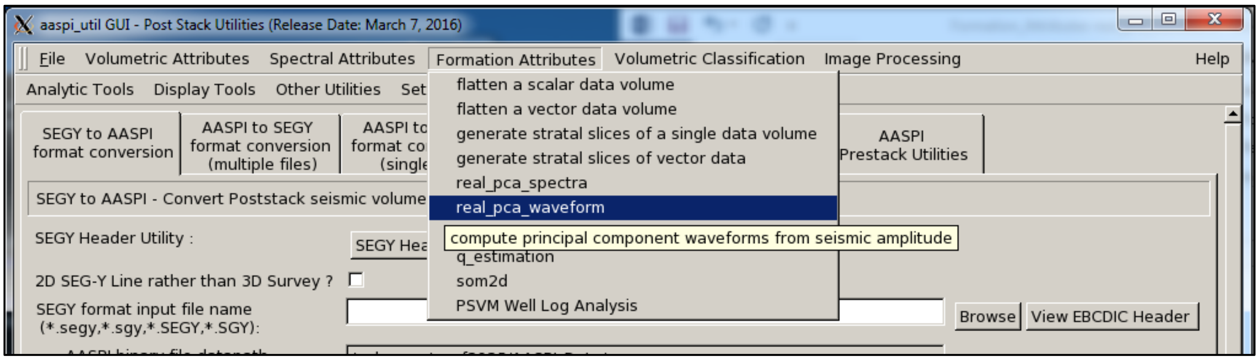
and so on for all N components. While the eigenvectors are orthogonal,

$$\mathbf{v_i} \cdot \mathbf{v_j} = \delta_{ij} , \quad (7)$$

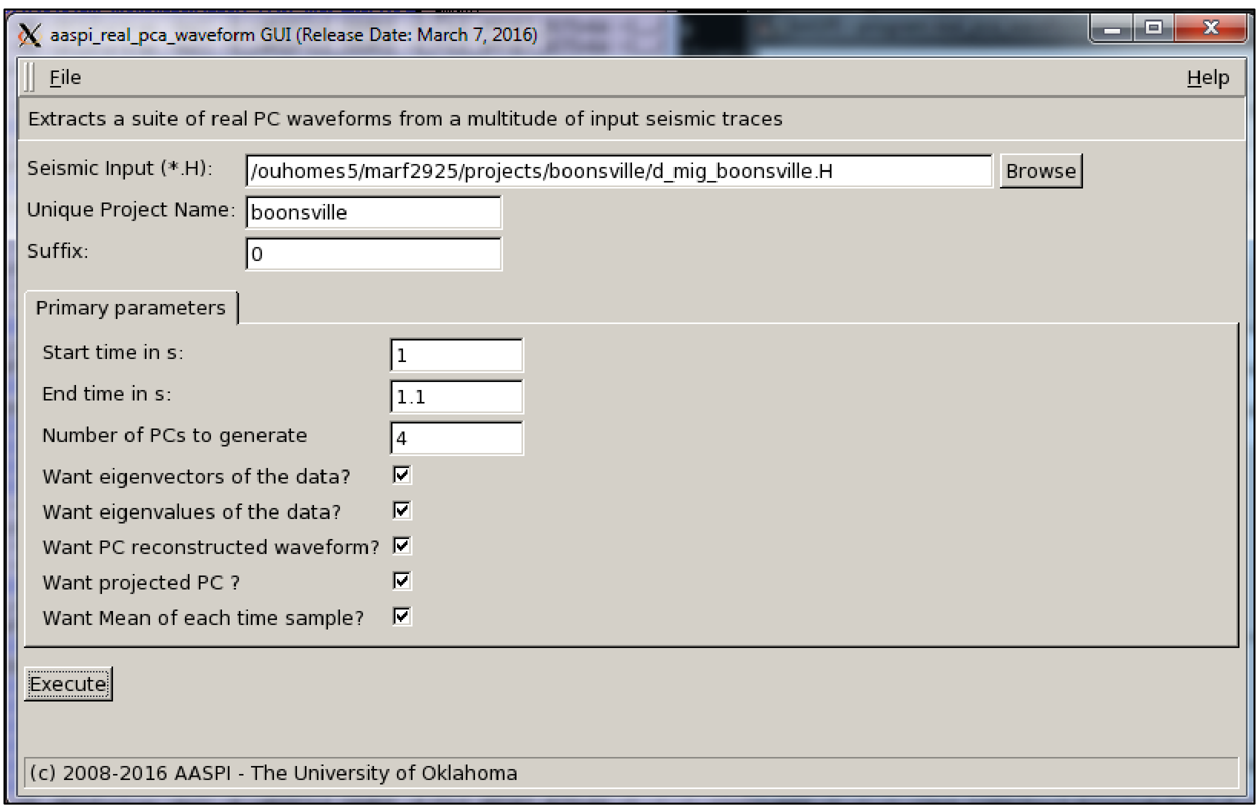
the principal components are not.

The pca_waveform_classification graphical user interface

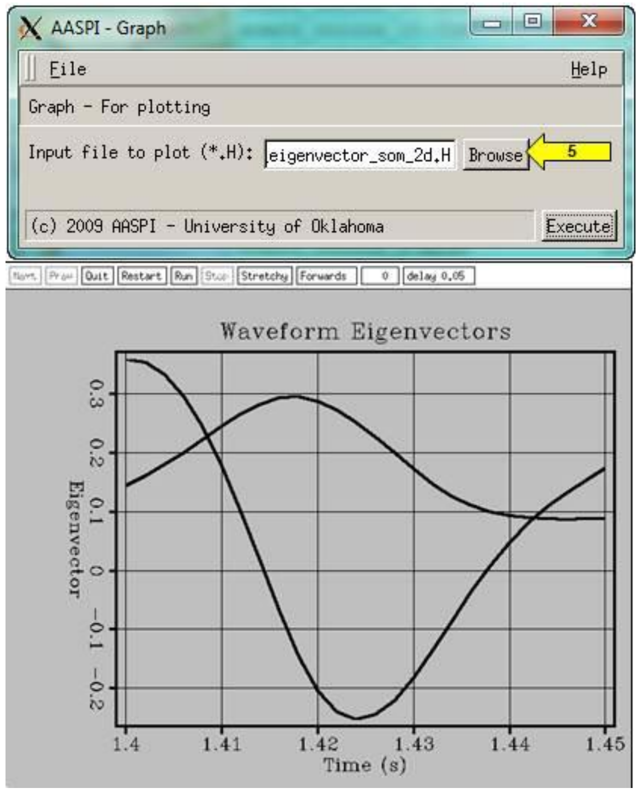
This Program **2D Facies Analysis** is launched from the *Formation Attributes* in the main **aaspi_util** GUI.



The following GUI appears:



To QC the outputs from **the pca_waveform_classification** program we can plot the eigenvectors, the eigenvalues and the means in the simple graph utility as shown above. The eigenvalues and the eigenvectors, which form the initial set or a priori training vectors, are shown below. The horizontal axis represents the samples of the waveform used in the analysis. The plot of the eigenvectors is the plot of the first two eigenvectors.



Crossplotting two or three principle components in Petrel

References