COMPUTING APPARENT CURVATURE – PROGRAM euler_curvature

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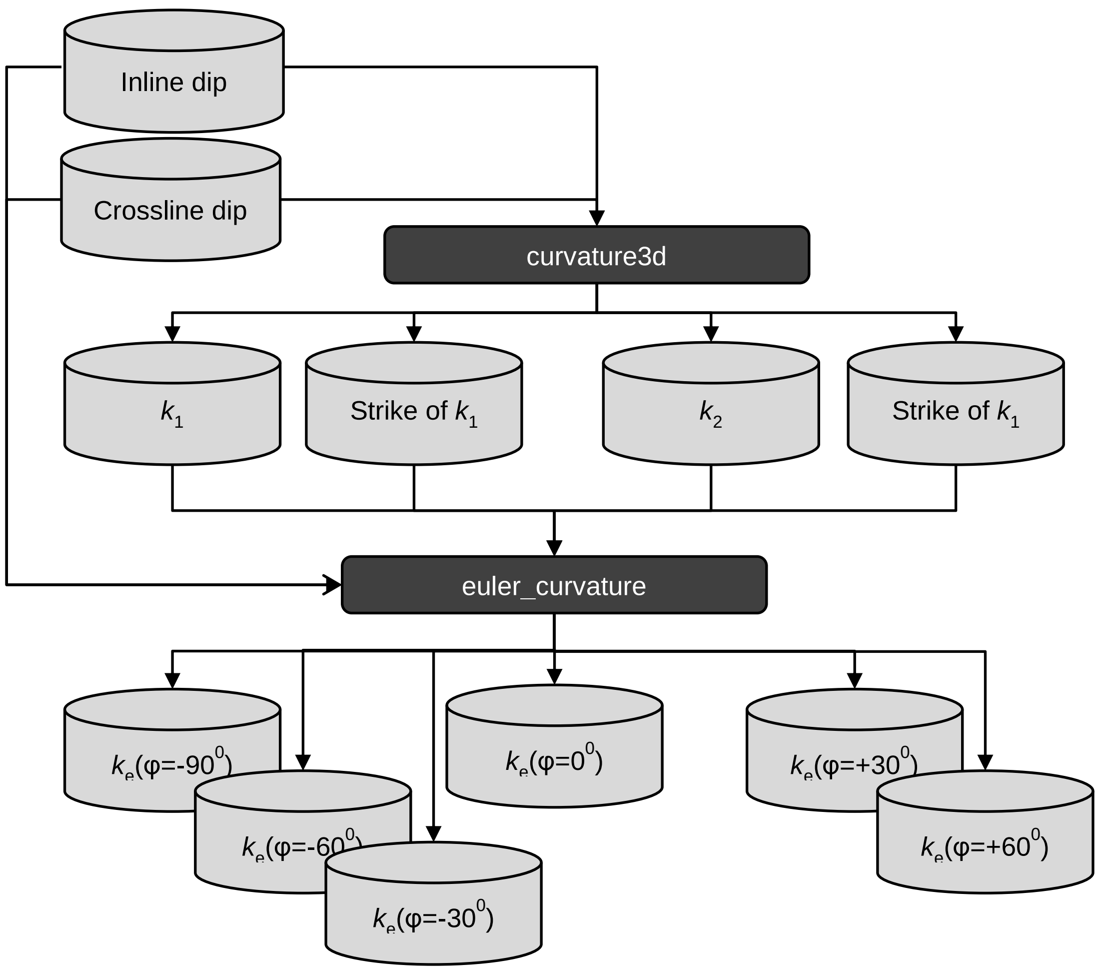
Overview

Euler curvature or apparent curvature in a user-defined direction provides a means of projecting the curvature matrix onto a vertical plane. Euler curvature provides many of the same advantages as shaded-relief maps, but because it is volumetric, can be applied to time slices in a volume rather than only to interpreted surfaces. The traditional strike component and dip components of curvature are Euler curvatures computed in the dip azimuth and strike directions of the locally dipping (and curved) surface.

Computation flow chart



Definition of Euler curvature orientation

Like all other AASPI programs, the azimuth ranges between -180° and +180° and therefore the apparent (Euler) curvature components, are defined clockwise from North, where North is at 0°, East at 90°, West at -90°, and South at ±180°. Note that the Euler curvature at azimuth φ is the negative of the Euler curvature at azimuth φ+180° so that typically you should only compute components between ±90

Output file naming convention

Program **euler_curvature** will always generate the following two output files:

Output file description	**File name syntax**
Program log information	euler_curvature_*unique_project_name_suffix*.log
Program error/completion information	euler_curvature_unique_project_name_suffix.err

where the values in red are defined by the program GUI. The errors we anticipated will be written to the **.err* file and be displayed in a pop-up window upon program termination. These errors, much of the input information, a description of intermediate variables, and any software trace-back errors will be contained in the **.log* file.

For structural curvature, the primary output files will have the form:

Output file description	**File name syntax**
Euler (apparent) curvature	k_euler_curvature_*unique_project_name_suffix_xxx*.log

whereas for amplitude curvature the primary output files will have the form:

Output file description	**File name syntax**
Euler (apparent) curvature	e_euler_curvature_*unique_project_name_suffix_xxx*.log

where the values indicated by xxx will be in degrees measured clockwise from North.

Theory of Euler Curvature

In 1767 the famous mathematician Leonhard Euler examined cross-sections of three-dimensional cylinders and realized that there were two principal curvatures, one which corresponded to the curvature of the circular cross section, and the other perpendicular to it that had an infinite radius, or zero curvature, and appeared as a straight line. For a quadratic surface, the two extreme (most-positive and most-negative principal) curvatures k_1 and k_2 are always orthogonal to each other and happen to form eigenvalue-eigenvector pairs locally defining the 3D surface. Honoring his work, the apparent curvature of any arbitrarily-oriented slice is now called “Euler curvature”.

Just as apparent dip (routinely used in interactive ‘sun-shading’ of picked horizons) can highlight subtle features of interest (e.g. Rijks and Jauffred, 1991) so can apparent, or Euler, curvature. The simplest way to envision Euler curvature is to envision a vertical slice striking at angle ψ from North through a fold. The intersection of the 3D fold with the vertical slice results in a 2D curve. Now, at any point on that curve, find the 2D circle that is tangent to it. The reciprocal of the radius of this 2D circle is the value of the Euler curvature in the vertical plane. Also note that one obtains the same circle whether examining the plane from left to right or right to left.

If (k_1, ψ_1) and (k_2, ψ_2) represent the magnitudes and strikes of the most-positive and most-negative principal curvatures, then the Euler curvature striking at an angle ψ' in the dipping plane tangent to the analysis point (where the vectors corresponding to ψ'_1 and ψ'_2 are orthogonal) is given as

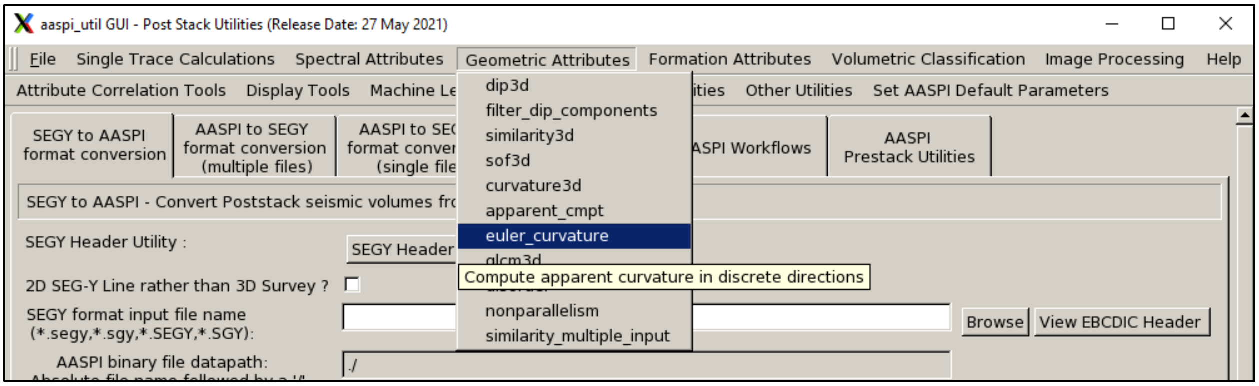
$$k_{\psi'} = k_1\cos^2(\psi' - \psi'_2) + k_2 \sin^2(\psi' - \psi'_2).$$

Note the squares over the cosine and sine term, which mathematically gives the same value of Euler curvature whether we look in the $+\psi'$ or $-\psi'$, from the most-negative principal curvature strike direction, ψ'_2. At this juncture the analogy to apparent dip (which would change sign) breaks down. While ψ'_1 *and* ψ'_2 will be perpendicular in the dipping plane tangent to the surface, the strikes projected onto the horizontal *x-y* plane, ψ_1 *and* ψ_2 , will not in general be perpendicular to each other. For program **euler_curvature**, we define the value of ψ for the entire volume along the horizontal *x-y* plane, project it onto to the dipping surface at each analysis point, apply equation 14.1, and compute $k_{\psi'}$. The dip of the local surface is defined by the inline and crossline dip components *p* and *q*. The algorithm computes a suite of Euler curvatures at azimuths ψ that are equally sampled in the *x-y* plane.

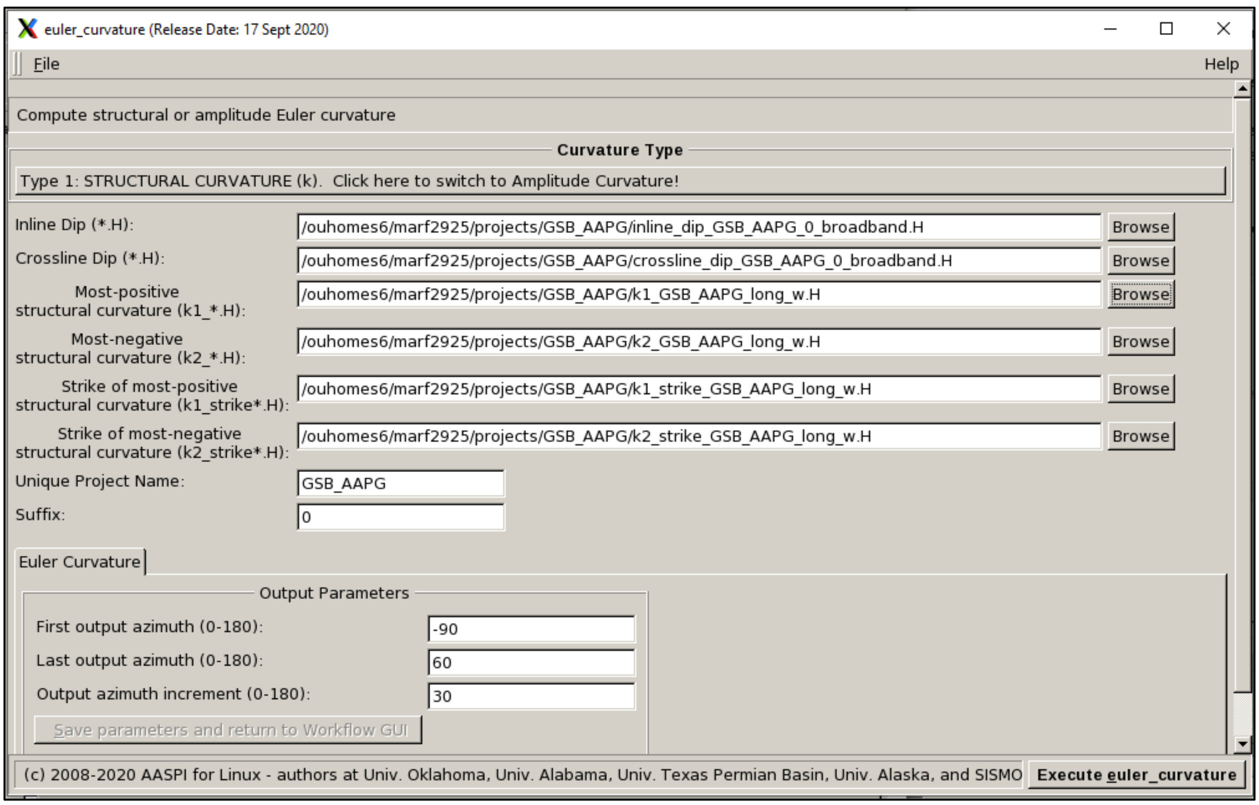
Tradition has it that Euler stumbled upon this formulation after his wife criticized him for preparing the family soup - she was unsatisfied with both circular cut and julienned or longitudinal cuts. With his mathematical genius he was able to cut the carrots at any arbitrary manner:

Program **euler_curvature** is found under the *Geometric Attributes* tab:

Invoking the euler_curvature GUI



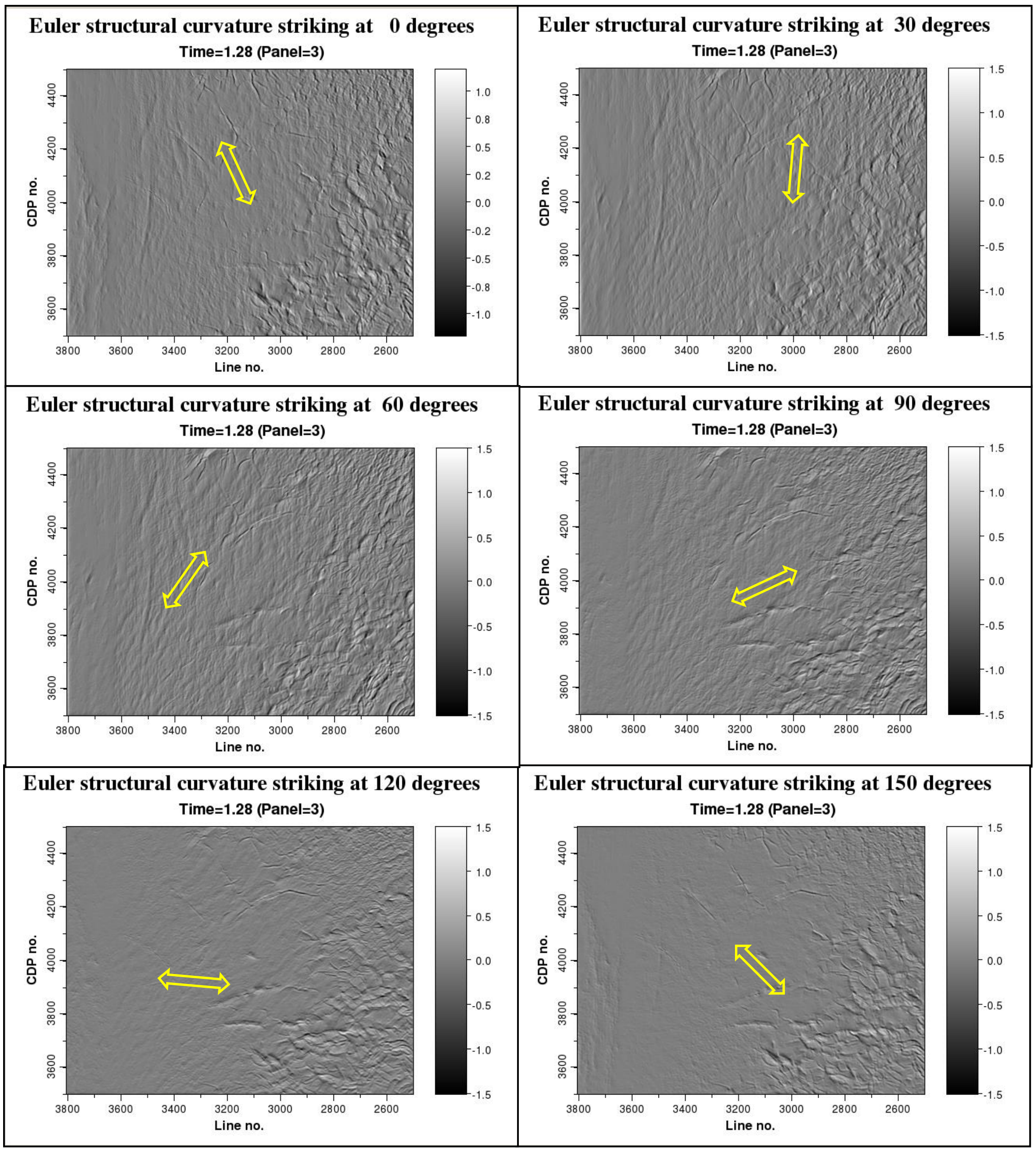
The algorithm can be run on either structural or amplitude curvature. Here we will use structural curvature for part of a New Zealand Great South Basin survey survey:



The following files were generated:

```
kmarfurt 6125 Sep 24 14:56 k_euler_curvature_GSB_AAPG_0__-90.H
kmarfurt 6125 Sep 24 14:56 k_euler_curvature_GSB_AAPG_0__-60.H
kmarfurt 6125 Sep 24 14:56 k_euler_curvature_GSB_AAPG_0__-30.H
kmarfurt 6125 Sep 24 14:56 k_euler_curvature_GSB_AAPG_0___60.H
kmarfurt 6125 Sep 24 14:56 k_euler_curvature_GSB_AAPG_0___30.H
kmarfurt 6125 Sep 24 14:56 k_euler_curvature_GSB_AAPG_0____0.H
```

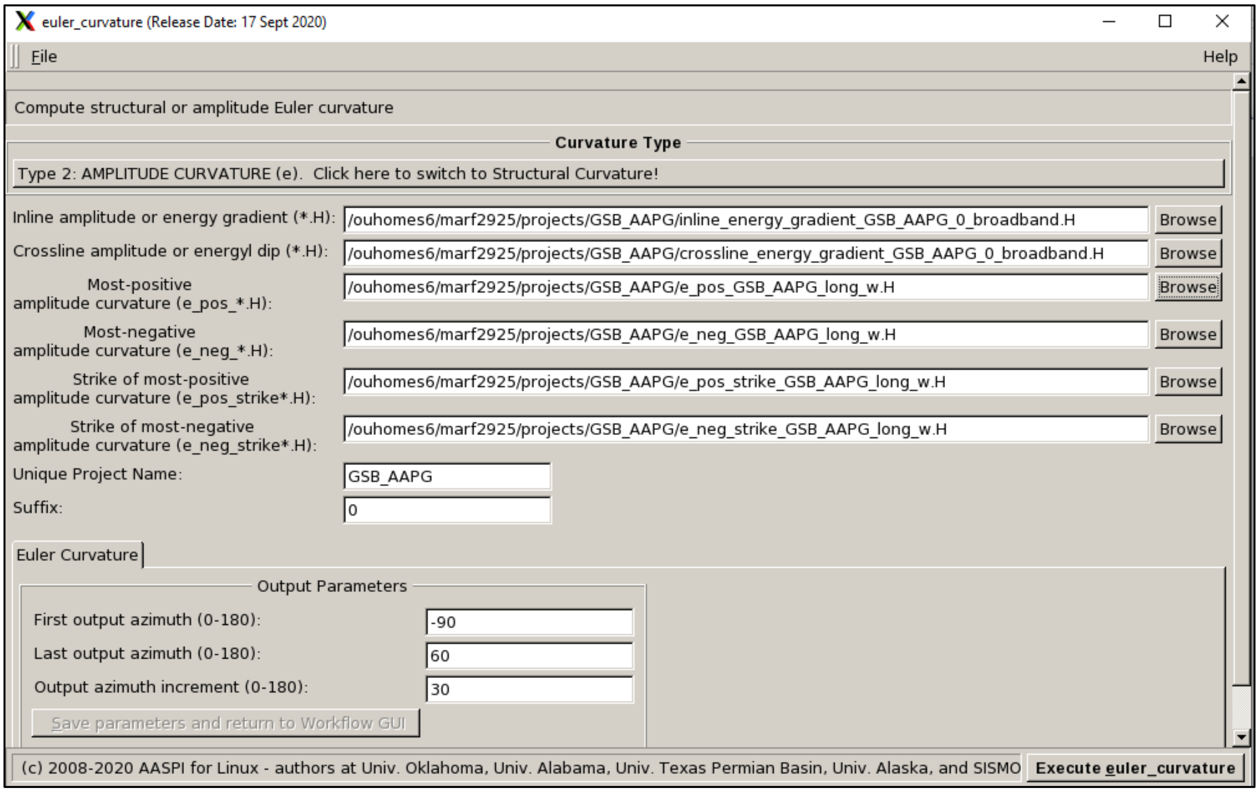
Time slices at t=1.280 s through structural Euler curvature volumes computed at 30^0 increments from the GSB survey look like such (see next page):



Recalling that the inline axis is rotated 25^0 from North reveals that the lineaments seen in the 30^0 component are approximately parallel to the inline axis, while those seen in the 120^0

component are approximately parallel to the crossline axis. In some parts of the world, such as the Marcellus Shale of Pennsylvania, natural fractures associated with subtle folds oriented in a specific direction (in that case perpendicular to the dominant NE-SW fold axes) are more easily hydraulically stimulated. For this reason, identifying zones where such subtle folding is more intense can be beneficial.

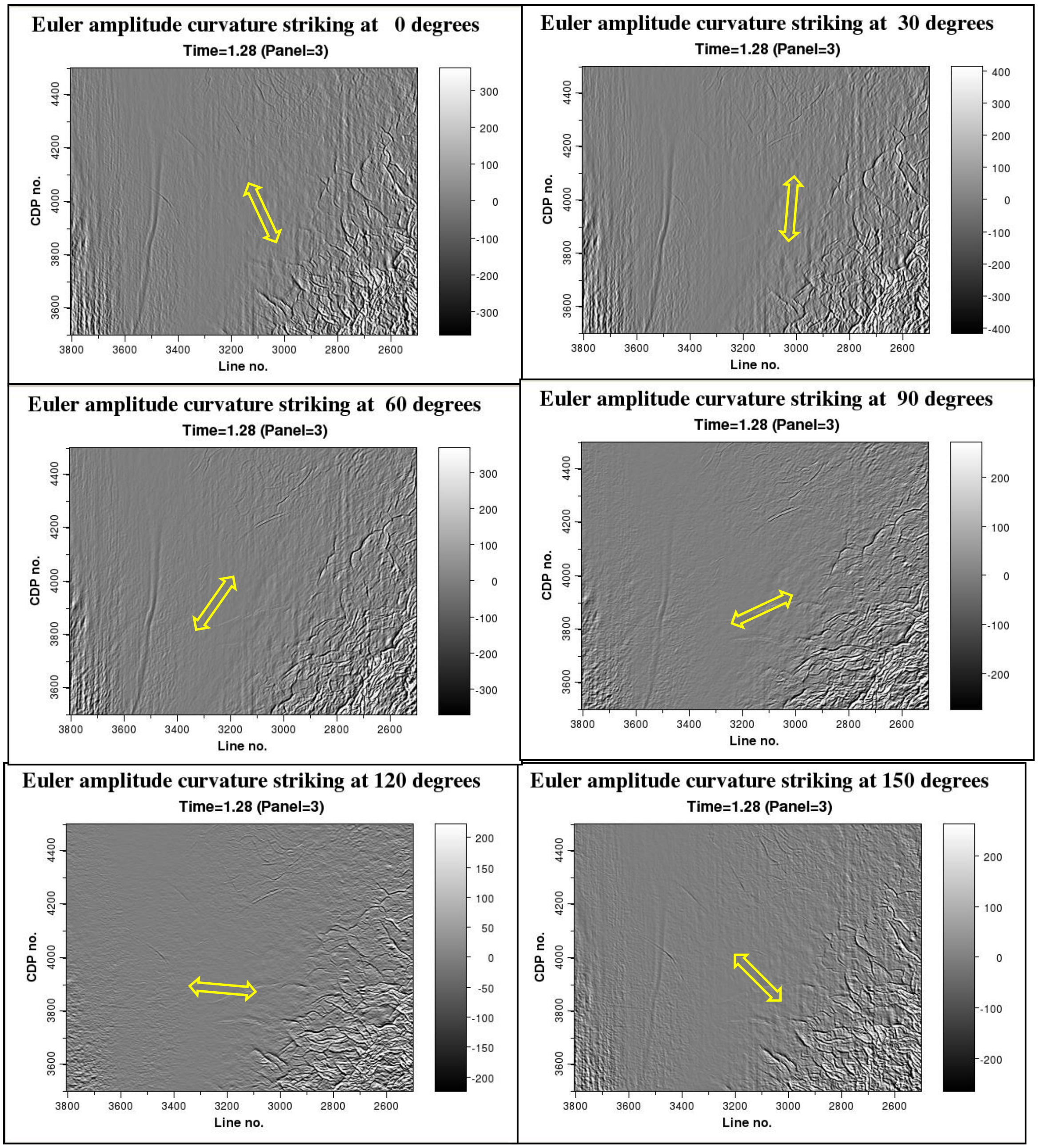
Computation of apparent (Euler) components of amplitude curvature are computed from e_{pos}, e_{neg}, and their corresponding strikes:



which generated the following files:

```
kmarfurt 6186 Sep 24 15:35 e_euler_curvature_GSB_AAPG_0__-90.H
kmarfurt 6186 Sep 24 15:35 e_euler_curvature_GSB_AAPG_0__-60.H
kmarfurt 6186 Sep 24 15:35 e_euler_curvature_GSB_AAPG_0__-30.H
kmarfurt 6186 Sep 24 15:35 e_euler_curvature_GSB_AAPG_0___30.H
kmarfurt 6186 Sep 24 15:35 e_euler_curvature_GSB_AAPG_0____0.H
kmarfurt 6186 Sep 24 15:35 e_euler_curvature_GSB_AAPG_0___60.H
```

and the corresponding apparent (Euler) amplitude curvature time slices:



References

Chopra, S., and K. J. Marfurt, 2011, Which curvature is right for you?, GCSSEPM 31st Annual Bob. F. Perkins Research Conference on Seismic attributes – New views on seismic imaging: Their use in exploration and production, 642-676.

Euler, L., 1767, Recherches sur la courbure des surfaces: Mémoires de l'académie des sciences de Berlin, v. E333, p. 119-143.