

Spectral Attributes: Program **spec_max_entropy**

MAXIMUM ENTROPY SPECTRAL ANALYSIS – PROGRAM spec_max_entropy

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Alternative Spectral Decomposition Algorithms

Spectral decomposition methods can be divided into three classes: those that use quadratic forms, those that use linear forms and those that use atom decomposition. Quadratic forms are based on Wigner-Ville distribution, which can be easily designed but lose the phase component of the data and therefore cannot be used in reconstruction. Linear forms are based on short time Fourier transform (STFT), and include the S transform and the continuous wavelet transform (or CWT) as in program **spec_cwt**. Program **spec_max_entropy** is a maximum entropy implementation of the short time Fourier transform. The program reconstructs the signal by using simple sines and cosines whose period can exceed that of the analysis window, using concepts described by Tad Ulrych, in his 2008 SEG distinguished lecture tour.

Computation flow chart

While there is only one input file to program **spec_max_entropy**, there are many output files which can easily fill your disk drives. Detailed spectral analysis will typically be done about a reservoir or other zone of interest, such that the input data volume may be windowed. If this is not the case, it is best to first run several of the statistical summary volumes including the peak magnitude, peak frequency, and peak phase, and phase residue of the entire volume. These

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spectra may be sufficient for your analysis. If not, the peak frequency volume will serve as a guide as to which spectral components are well tuned.

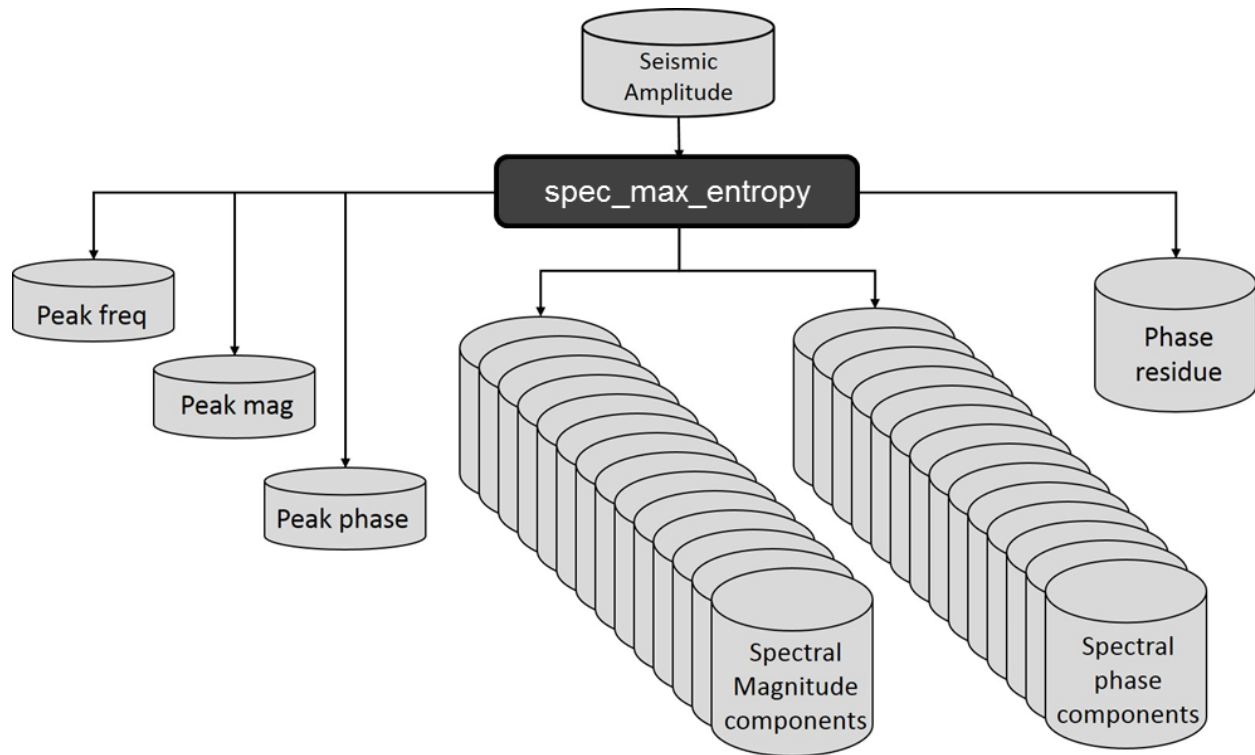
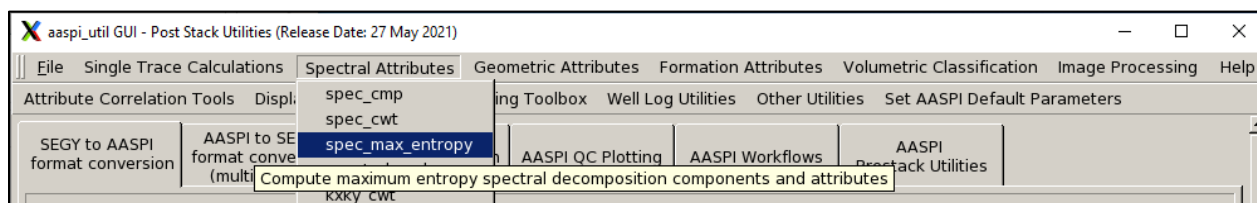


Figure 1. The flow chart for program **spec_max_entropy** – maximum entropy spectral analysis.

Computing spectral components

To begin, click the *Volumetric Attributes* tab in the **aaspi_util** window and select the program **spec_max_entropy**:



Program **spec_max_entropy** performs spectral decomposition using a maximum entropy method. The following window appears (see next page):

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The screenshot shows the 'aaspi_spec_max_entropy GUI' window. At the top, it says 'Decompose the input seismic into time-frequency spectral components using the Constrained Least Squares Spectral Analysis algorithm'. Below this, there are input fields for 'Seismic Input (*.H):' (with a file path and a 'Browse' button), 'Unique Project Name:' (set to 'boonsville'), 'Suffix:' (set to '0'), and a 'Verbose:' checkbox. A yellow arrow labeled '1' points to the 'Seismic Input' field. Below these are three tabs: 'Primary Parameters', 'Define Output Subvolume Limits', and 'Parallelization parameters'. The 'Primary Parameters' tab is active, showing 'Spectral Parameters' and 'Entropy Parameters'. Under 'Spectral Parameters', there are frequency inputs f1 (5), f2 (10), f3 (90), and f4 (120), all in cycles/s. A yellow arrow labeled '2' points to this section. Under 'Entropy Parameters', there are 'Window Length (s):' (0.04), 'Number of iterations:' (2), 'alpha scaling parameter:' (0.001), and 'Phase residue threshold (Percent):' (5). Yellow arrows labeled '3', '4', '5', and '6' point to these respective fields. Below these is the 'Output spectral components' section with 'Lowest output frequency, f_low (cycles/unit1):' (5), 'Highest output frequency, f_high (cycles/unit1):' (100), and 'Frequency Increment: (cycles/s)' (5). At the bottom, the 'Results' section has several checkboxes: 'Output peak attributes?' (checked), 'Output spec mag cmpt?' (checked), 'Output spec phase cmpt?' (checked), 'Output spec voice cmpt?' (checked), 'Compute phase residue attributes?' (checked), and 'Want reconstructed data?' (checked). A yellow arrow labeled '7' points to the 'Output spec phase cmpt?' checkbox. Below the checkboxes is the 'Output component file format:' section with two radio buttons: 'Multiple files - one per frequency component' (selected) and 'A single hypercube file - with frequency as axis 2'. At the very bottom, there is a footer with copyright information and an 'Execute spec_max_entropy' button.

You will note that many of the attributes appear to be similar to those of program **spec_cmp**, spectral decomposition using a complex matching pursuit algorithm. First, enter the (1) the name of the *Seismic Input (*.H)* file you wish to decompose, as well as a *Unique Project Name* and *Suffix* as you have done for other AASPI programs.

Maximum entropy spectral analysis (**max_entropy**)

When using discrete Fourier transforms, the analysis window length implicitly defines the maximum period that can be analyzed, such that if we defined the window to be 40 ms long, the lowest frequency would be 25 Hz. In his distinguished lecture, Ulrych (2008) discussed computing the number of years that it would take for Neptune to orbit the sun. If we made measurements every day of our life (say for 85 years) we would still not have a full cycle. He then emphasized that the part of the cycle we did not measure (between 85 and 164.79 years) is not zero, but rather unknown. In this simple example, maximum entropy asks which sinusoidal period best represents the measurements we have made on Neptune in a least-

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squares sense. In this manner, we compute a period larger than that the length of the analysis window. If we wished to measure the Milankovitch cycles, we would first obtain the largest cycle, fit it to the measurements, subtract it from the measurements, and then repeat the process for the 2nd most important cycle. This process is probably familiar to those who use high resolution discrete Radon transforms, developed by Ulrych's students and grad-students.

This approach results in spectra that have reduced window smearing for a given window length relative to the discrete Fourier transform irrespective of window shape, and a time-frequency analysis with a combination of time and frequency resolution that is superior to the short time Fourier transform and the continuous wavelet transform. As compared with the continuous wavelet transform, the method has improved temporal resolution, particularly at low frequencies.

Several investigators have used different empirical criteria to define inversion-based spectral analysis methods. Most notably, Portniaguine and Castagna (2004) approach the seismic wavelet decomposition problem by inverting the normal equations using an iteratively reweighted least-squares regularization algorithm. Further implementation of this algorithm is described by Puryear et al. (2010) that they call constrained least-squares spectral analysis. In both cases, the objective function is the same – to fit the data with the minimum number of sines and cosines.

Theory: Maximum entropy spectral decomposition

Start with the definition of the forward problem.

$$\mathbf{F}\mathbf{m} = \mathbf{d} , \quad (\text{A1})$$

where \mathbf{F} is the kernel matrix composed of complex sinusoidal basis functions,

$$F_{nk} = \exp[i(2\pi k \Delta f n \Delta t)] = \cos(2\pi k \Delta f n \Delta t) + i \sin(2\pi k \Delta f n \Delta t) , \quad (\text{A2})$$

$m_k = m(2\pi k \Delta f)$ is the model parameter vector (i.e. the desired spectral coefficients), and $d_n = d(n \Delta t)$ is the windowed seismic amplitude data. Forming the normal equations by multiplying both sides of equation A1 by \mathbf{F}^H , the complex conjugate or Hilbert transpose of \mathbf{F} , gives

$$\mathbf{F}^H \mathbf{F} \mathbf{m} = \mathbf{F}^H \mathbf{d} . \quad (\text{A3})$$

The objective of the inverse problem is to compute \mathbf{m} given \mathbf{F} and \mathbf{d} . The classical least-squares solution to equation A1 is then

$$\mathbf{m} = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{d} . \quad (\text{A4})$$

Conventional Fourier transforms used in filtering are constructed to be orthogonal, such that $\mathbf{F}^H \mathbf{F} = \mathbf{I}$, where \mathbf{I} is the identity matrix. In this case, equation A4 reduces to

$$\mathbf{m} = \mathbf{F}^H \mathbf{d} , \quad (\text{A5})$$

which is equivalent to the discrete Fourier transform of the windowed trace segment. If the analysis window is $T=0.1$ s, orthogonality requires $\Delta f=1/T=10$ Hz. If we wish to estimate intermediate spectral components at a 2 Hz interval, we need to use the more general equation A4. A finer frequency increment results an underdetermined system with more unknowns than equations. We constrain the inversion by introducing two diagonal matrices. The data weight matrix, \mathbf{W}_d , acts as a taper, where smaller weights at the window edges provides for greater tolerance of the misfit. In contrast, the model weight matrix, \mathbf{W}_m , change iteratively, in general favoring the strongest spectral components, and attenuating the corresponding weaker aliases. Applying \mathbf{W}_m and \mathbf{W}_d to equation A1,

$$\mathbf{W}_d \mathbf{F} \mathbf{W}_m (\mathbf{W}_m)^{-1} \mathbf{m} = \mathbf{W}_d \mathbf{d} , \quad (\text{A6})$$

we introduce the weighted quantities

$$\mathbf{F}_w = \mathbf{W}_d \mathbf{F} \mathbf{W}_m \text{ and } \mathbf{m}_w = \mathbf{W}_m^{-1} \mathbf{m} , \quad (\text{A6})$$

and then recast equation A5 as a model and data-weighted ill-posed inverse problem:

$$\mathbf{F}_w \mathbf{m}_w = \mathbf{W}_d \mathbf{d} . \quad (\text{A7})$$

To solve equation A7, we add regularization:

$$\min \mathcal{E}^2 = \|\mathbf{F}_w \mathbf{m}_w - \mathbf{W}_d \mathbf{d}\|^2 + \alpha \|\mathbf{m}_w\|^2 \quad (\text{A8})$$

where α is a regularization parameter that can be varied to control the sparsity and stability of the solution. Note that if \mathbf{W}_m is proportional to \mathbf{m} , small values of \mathbf{m} become large values of \mathbf{m}_w which are then minimized, giving us a sparse solution. Using an analytical Lagrange solution to equation A8, we solve first for the weighted model parameters

$$\mathbf{m}_w = \mathbf{F}_w^H (\mathbf{F}_w \mathbf{F}_w^H + \alpha \mathbf{I})^{-1} \mathbf{W}_d \mathbf{d} , \quad (\text{A9})$$

followed by reconstruction

$$\mathbf{m} = \mathbf{W}_m \mathbf{m}_w . \quad (\text{A10})$$

Values of α are chosen to be a percentage of $\|\mathbf{F}_w \mathbf{F}_w^H\|$. Portniaguine and Castagna (2004) find the CLSSA utilizing a real basis function to be not quite as good as using a complex basis function, but better than the conventional DFT. This is significant because CLSSA using only the real waveform has precisely the same temporal resolution as the Fourier transform while having greatly improved frequency resolution. This indicates that, for a given window, CLSSA has a better Heisenberg uncertainty product (standard deviation of the waveform in time multiplied by the standard deviation of the spectrum) than does the Fourier transform. Thus far, they have not found a window for which this is not the case.

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The *Reconstruction parameters* (2) indicates the four corner frequencies f_1 , f_2 , f_3 , and f_4 used to reconstruct the seismic data after flattening. A raised-cosine taper is applied between f_1 and f_2 as well as between f_3 and f_4 . The choice of frequency increment and range does not significantly impact the decomposition time, only the output size of the data if you choose to output either spectral magnitude or spectral phase components as either a sequence of constant frequency volumes or as an equivalent 4D volume. In this example we used a 40 ms window, which implies a frequency resolution of $\Delta f = 1/.040 \text{ s} = 25 \text{ Hz}$ is necessary to reconstruct the data using a DFT. Using this argument, we can say that the spectra is oversampled, which is validated by the excellent reconstruction on the image below.

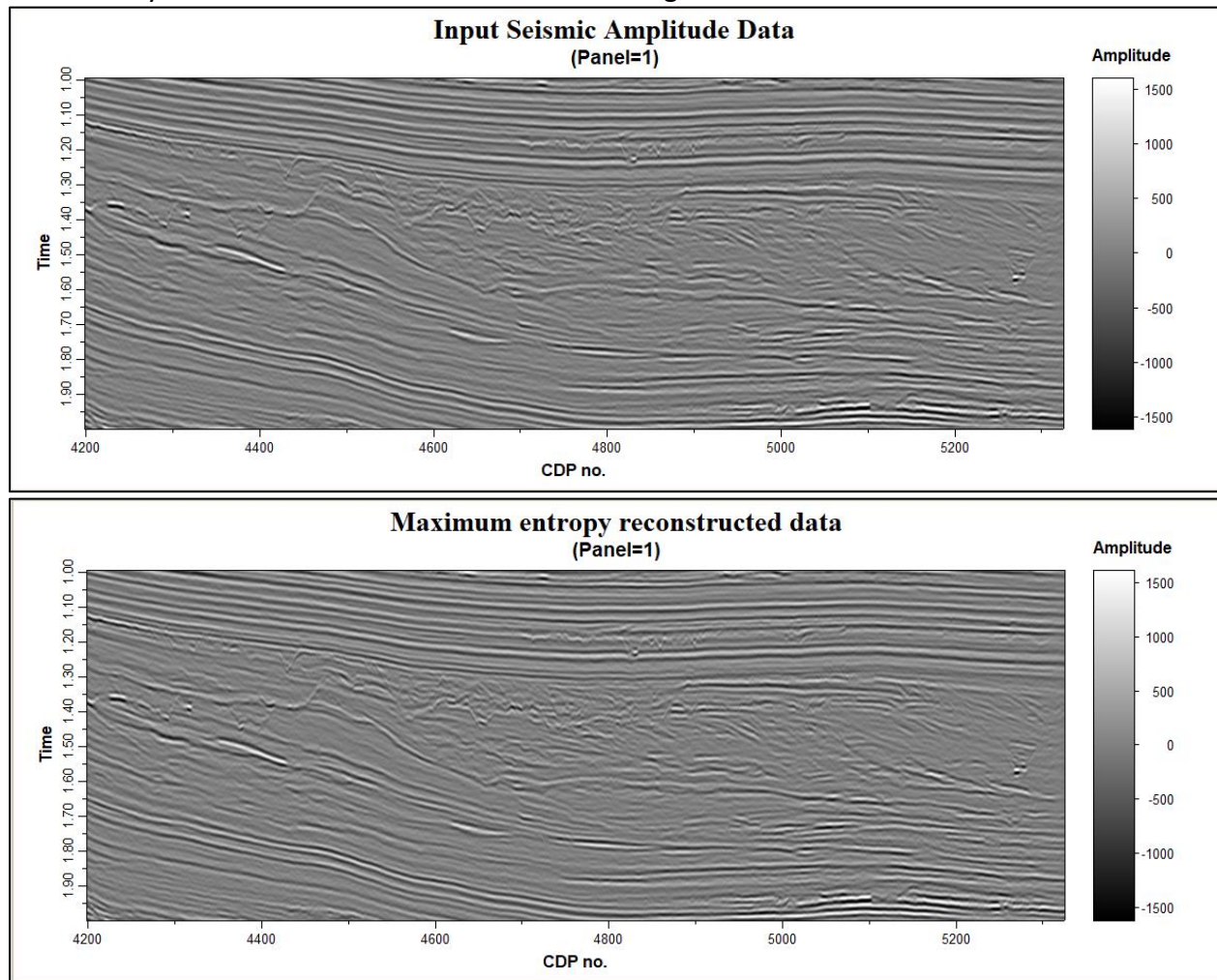


Figure 2.

The (3) *Window length* indicates the length of the time window function used in spectral decomposition. As discussed above, the maximum entropy algorithm is focused on solving an inverse problem. The window length parameter determines how long the inverse operator would cover, which can affect the time-frequency resolution. As an iterative method, the (4) *Number of iterations* defines the number of iterations used in iteratively reweighted least-squares solution. A larger number of iterations provides a more-accurate least-squares

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approximation, but also consumes more time. The (5) *alpha scaling parameter* is the regularization parameter α in equations A8 and A9 above, which stabilizes the least-squares regression process.

The (6) *Phase residue threshold (Percent)* limits the phase residue attribute computation. When it is larger, more phase residue will appear corresponding to weaker events.

Theory: The Phase Residue

Ghiglia and Pritt (1998) provide an excellent survey of 2D phase-unwrapping techniques and show how a complex residue theorem based on vector calculus can be applied to the phase-unwrapping problem. They use a rectangular integration path aligned with the t and f axes. We choose a smaller diamond-shaped integration path about each sample ($j\Delta t, k\Delta f$) given by:

$$I_{jk} = \frac{W[\psi(t_{j-1}, f_k) - \psi(t_j, f_{k-1})]}{2\pi} + \frac{W[\psi(t_j, f_{k-1}) - \psi(t_{j+1}, f_k)]}{2\pi} + \frac{W[\psi(t_{j+1}, f_k) - \psi(t_j, f_{k+1})]}{2\pi} + \frac{W[\psi(t_j, f_{k+1}) - \psi(t_{j-1}, f_k)]}{2\pi} \quad (B1)$$

where ψ is the phase and W is a wrapping operator that produces an output that falls between $\pm\pi$. If the integral I_{jk} in equation B1 is nonzero, there are inconsistent phase points, which Ghiglia and Pritt (1998) call “phase residues”. The figure below shows how the residue is calculated for a small portion of a typical wrapped time-frequency phase matrix. Area A is continuous, with a phase residue = 0.0, while area B is discontinuous with a phase residue = 1.0.

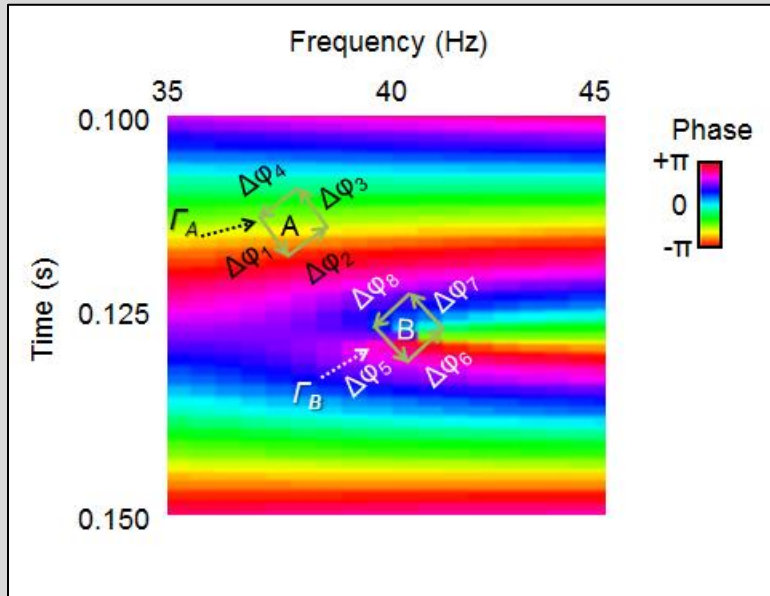


Figure 3. Line integrals about analysis points A and B used to compute the phase residue.

Matos et al. (2011) find phase attributes are sensitive to the same kinds of stratigraphic discontinuities seen by analyzing the magnitude component of time-frequency distribution using wavelet transforms and the continuous wavelet transform. Because phase is often a more seismic measure than magnitude, it holds significant promise in mapping stratigraphic unconformities.

Accurate computation of phase residues requires relatively fine sampling across frequency. We suggest setting the computational $\Delta f=1$ Hz. Unfortunately, if you do this and choose to output these many components as well, you may fill up your disk drive. To analyze individual components we suggest a coarser output $\Delta f=5$ Hz for subsequent loading into your interpretation workstation.

Statistical measures of the spectrum

Like program `spec_cmp`, program `spec_max_entropy` provides several statistical measures of the spectrum that can be used in addition to or in place of the full 4D spectral components. The peak spectral magnitude, peak spectral frequency, and peak spectral phase are easy to understand. You obtain these by placing a checkmark in front of *Want peak attributes?*

Theory: Statistical measures of the spectrum

Gaussian statistics such as the mean, standard deviation, kurtosis, and skewness are sometimes used to represent a seismic spectrum, with the mean representing the average spectrum, the standard deviation the bandwidth, and kurtosis and skewness deviations from the Gaussian spectrum model. Unfortunately, seismic processors try as hard as they can to make the spectra flat, which is decidedly non-Gaussian. Zhang (2010) therefore constructed a suite of attributes that better define these kinds of spectra. The local bandwidth is defined as the difference between user-defined percentiles. The range-trimmed mean is simply the average frequency within these percentiles. The slope is a measure of how the spectrum changes with frequency – e.g. increasing, flat, or decreasing. Finally, the roughness is a measure of local smoothness of the spectrum.

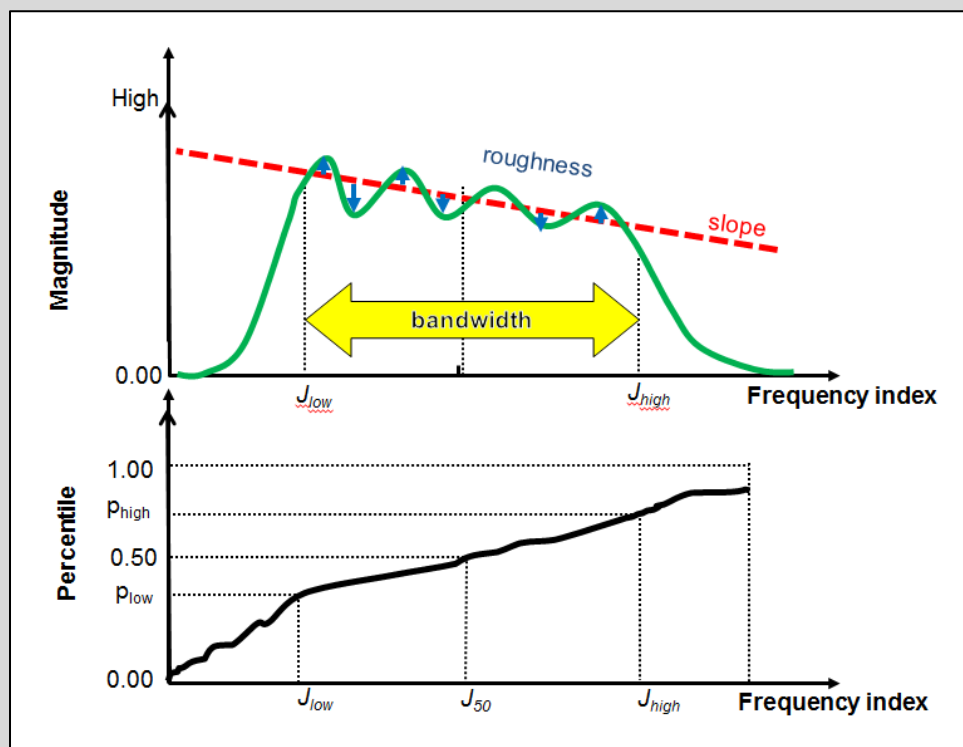
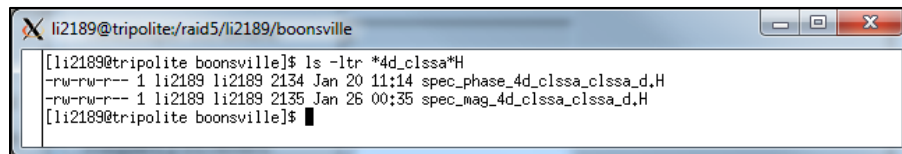


Figure 4. The shape of a spectrum (in green) containing 50 or more components approximated by average frequency, bandwidth, slope, and roughness attributes.

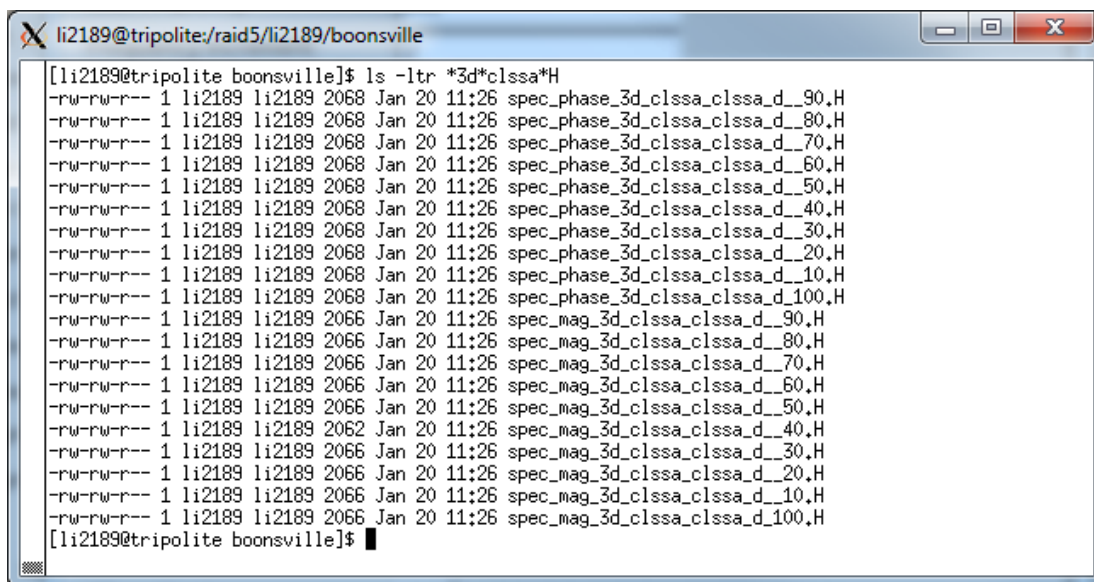
Individual spectral components

If you place a checkmark in front of *Want spec mag cmpt?* or *Want spec phase cmpt?* you will obtain each of the spectral components that range between f_1 and f_4 with the desired *Frequency Increment*. For interpretation of the components on most interpretation workstations, it may be easier to load these components separately. If you place a check mark in front of (9) *Store cmpts as 4D cubes?*, you obtain spectral gathers that are ordered with the time axis running fastest, followed by the frequency axis, (such that the first two indices represent a time-frequency distribution) followed by the CDP numbers (inline axis) and line numbers (crossline axis). The 4D volumes will have the following names for this job:



```
li2189@tripolite:/raid5/li2189/boonsville
[li2189@tripolite boonsville]$ ls -ltr *4d_clssa*H
-rw-rw-r-- 1 li2189 li2189 2134 Jan 20 11:14 spec_phase_4d_clssa_clssa_d,H
-rw-rw-r-- 1 li2189 li2189 2135 Jan 26 00:35 spec_mag_4d_clssa_clssa_d,H
[li2189@tripolite boonsville]$
```

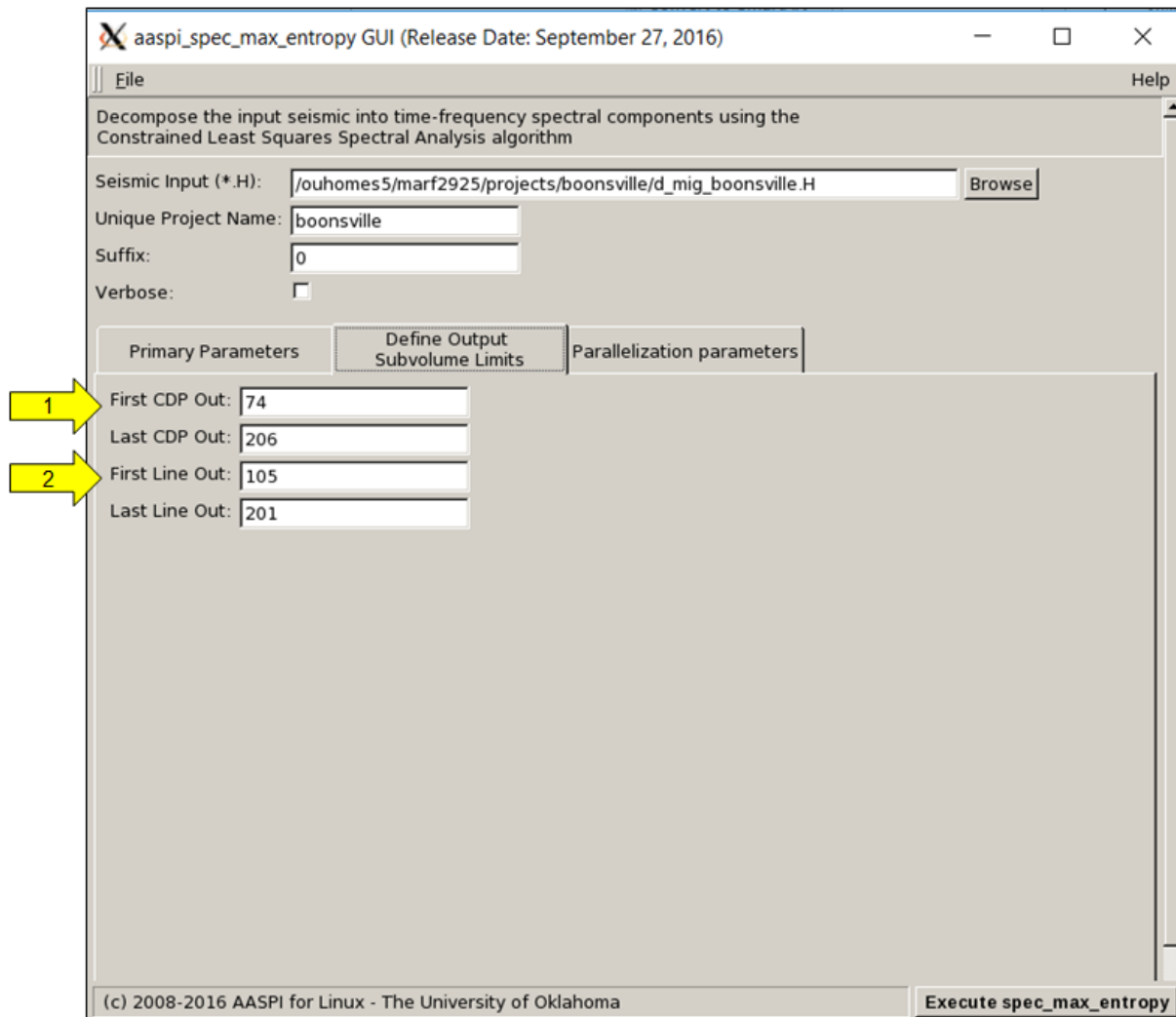
If you ask for spectral components not to be stored as a 4D cube the constant-frequency 3D spectral magnitude and spectral phase volumes will have the frequency value encoded in the file name:



```
li2189@tripolite:/raid5/li2189/boonsville
[li2189@tripolite boonsville]$ ls -ltr *3d*clssa*H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__90,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__80,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__70,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__60,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__50,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__40,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__30,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__20,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d__10,H
-rw-rw-r-- 1 li2189 li2189 2068 Jan 20 11:26 spec_phase_3d_clssa_clssa_d_100,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__90,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__80,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__70,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__60,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__50,H
-rw-rw-r-- 1 li2189 li2189 2062 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__40,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__30,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__20,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d__10,H
-rw-rw-r-- 1 li2189 li2189 2066 Jan 20 11:26 spec_mag_3d_clssa_clssa_d_100,H
[li2189@tripolite boonsville]$
```

There are several 'expert' controls under the *Extended* tab:

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Since the amount of output can be quite large, it may be useful to run **spec_max_entropy** on only a limited range of (1) cdp's (cross line indices) and (2) lines (inline indices).

You will probably want to experiment with these parameters a bit to calibrate them for the kind of data you encounter. It is reasonable to expect that most surveys of a similar vintage from the Gulf of Mexico will have similar spectral ranges and signal-to-noise ratios. Likewise, similar surveys acquired in the Permian Basin of west Texas will be similar to each other. In order to simplify parameter choices, you can then *cat* AASPI ".parms" file to examine its content or set it to your desired default parameters:

Spectral Attributes: Program **spec_max_entropy**

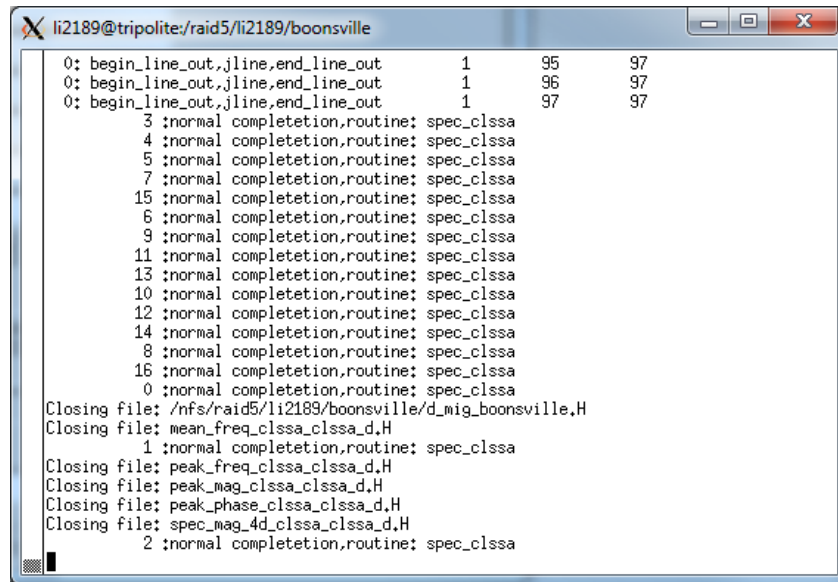


```
li2189@tripolite:/raid5/li2189/boonsville
[li2189@tripolite boonsville]$ cat spec_clssa.parms
input_fn="/nfs/raid5/li2189/boonsville/d_mig_boonsville,H"
inline_dip_fn=""
crossline_dip_fn=""
error_fn="spec_clssa_clssa_d,err"
output_fn="spec_clssa_clssa_d,out"
avg_spec_power_fn="avg_spec_power_clssa_clssa_d,H"
avg_spec_power_scale_fn="avg_spec_power_scale_clssa_clssa_d,H"
avg_spec_unwrapped_phase_fn="avg_unwrapped_phase_clssa_clssa_d,H"
modeled_fn=""
residual_fn=""
d_recon_fn=""
spec_mag_3d_fn=""
spec_mag_4d_fn="spec_mag_4d_clssa_clssa_d,H"
spec_phase_3d_fn=""
spec_phase_4d_fn=""
spec_voice_3d_fn=""
spec_voice_4d_fn="spec_voice_4d_clssa_clssa_d,H"
phase_residue_mag_fn=""
phase_residue_freq_fn=""
peak_mag_fn="peak_mag_clssa_clssa_d,H"
peak_mag_above_avg_fn="peak_mag_above_avg_clssa_clssa_d,H"
peak_freq_fn="peak_freq_clssa_clssa_d,H"
peak_phase_fn="peak_phase_clssa_clssa_d,H"
mean_freq_fn="mean_freq_clssa_clssa_d,H"
rtm_mag_fn="rtm_mag_clssa_clssa_d,H"
bandwidth_fn="bandwidth_clssa_clssa_d,H"
slope_fn="slope_clssa_clssa_d,H"
roughness_fn="roughness_clssa_clssa_d,H"
unique_project_name="clssa"
suffix="d"
f1=5
f2=10
f3=120
f4=140
df_out=2
win_len=40
n_iter=2
alpha_f=0.001
residue_threshold=5
want_peak_attr=y
want_spectral_mag=y
want_spectral_phase=n
want_phase_residue=n
want_reconstructed_data=n
want_4d_cubes=y
use_mpi=y
processors_per_node=16
node_list="localhost"
verbose=n
first_line_out=105
last_line_out=201
first_cdp_out=74
last_cdp_out=206
[li2189@tripolite boonsville]$
```

The file in your working directory will always take precedence over the one in the `${AASPIHOME}/scripts` directory.

As in all the AASPI GUIs, click *Execute* to run the program. The end of your run should look something like the following:

Spectral Attributes: Program **spec_max_entropy**



```
li2189@tripolite:/raid5/li2189/boonsville
0: begin_line_out,jline,end_line_out      1      95      97
0: begin_line_out,jline,end_line_out      1      96      97
0: begin_line_out,jline,end_line_out      1      97      97
  3 :normal completion,routine: spec_clssa
  4 :normal completion,routine: spec_clssa
  5 :normal completion,routine: spec_clssa
  7 :normal completion,routine: spec_clssa
 15 :normal completion,routine: spec_clssa
  6 :normal completion,routine: spec_clssa
  9 :normal completion,routine: spec_clssa
 11 :normal completion,routine: spec_clssa
 13 :normal completion,routine: spec_clssa
 10 :normal completion,routine: spec_clssa
 12 :normal completion,routine: spec_clssa
 14 :normal completion,routine: spec_clssa
  8 :normal completion,routine: spec_clssa
 16 :normal completion,routine: spec_clssa
  0 :normal completion,routine: spec_clssa
Closing file: /nfs/raid5/li2189/boonsville/d_mig_boonsville,H
Closing file: mean_freq_clssa_clssa_d,H
  1 :normal completion,routine: spec_clssa
Closing file: peak_freq_clssa_clssa_d,H
Closing file: peak_mag_clssa_clssa_d,H
Closing file: peak_phase_clssa_clssa_d,H
Closing file: spec_mag_4d_clssa_clssa_d,H
  2 :normal completion,routine: spec_clssa
```

Example 1: Comparison of the three decomposition algorithms

Our first example compares vertical slices of spectral components at 20 Hz and 50 Hz through the UT BEG Stratton data volume. Note that 20 Hz has a period of 0.050 s which is larger than the 0.040 s analysis window used in **spec_max_entropy**. The 20 Hz component from **spec_cwt** is not really a component, but rather the relatively broad band response of a Morlet wavelet centered at 20 Hz, with a Gaussian spectrum with half power = f. The 20 Hz component from **spec_cmp** is the complex sum of all 20 Hz components of Morlet wavelets about a given time sample. For this reason, both **spec_max_entropy** and **spec_cmp** have higher frequency resolution than **spec_cwt**.

Spectral Attributes: Program `spec_max_entropy`

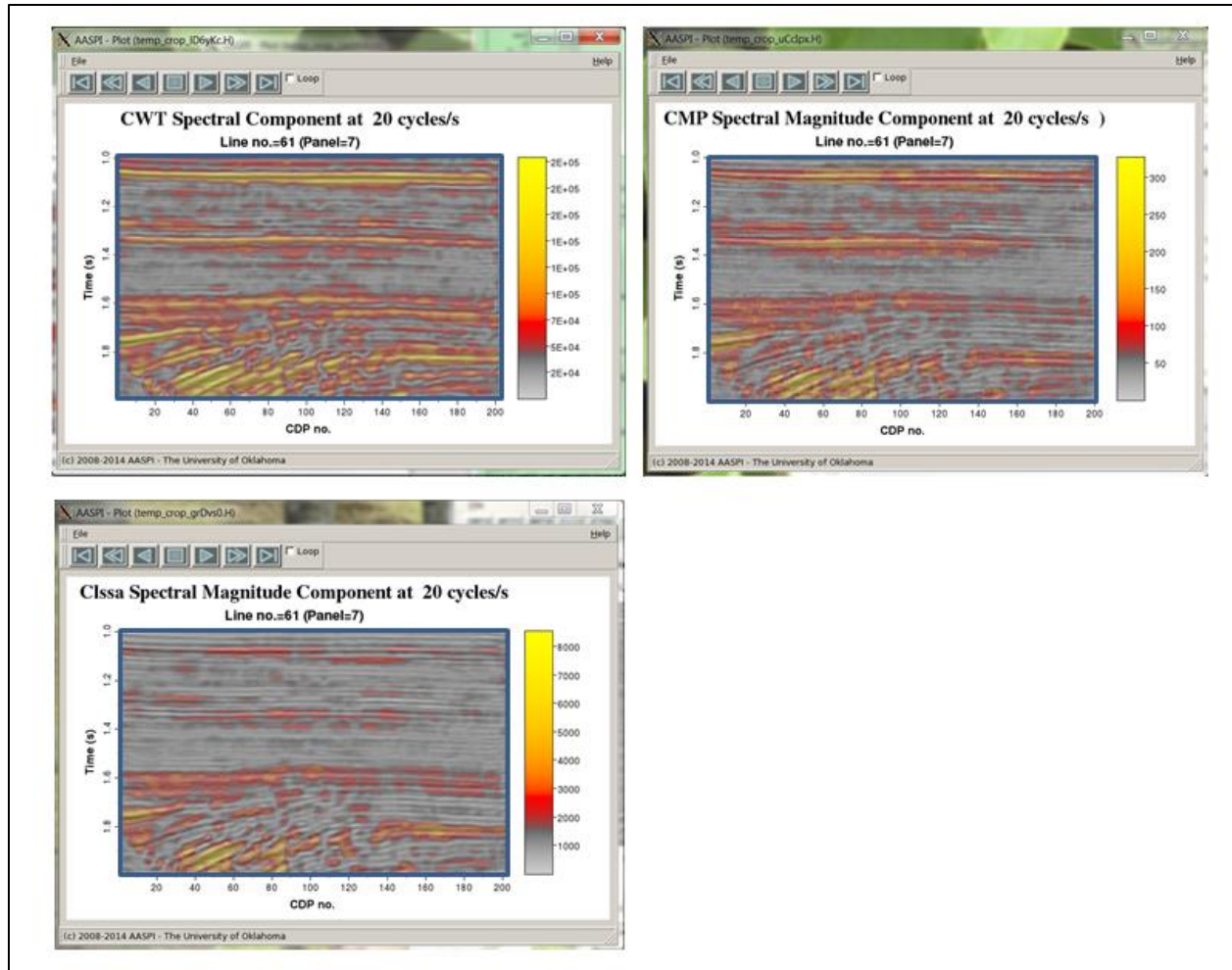


Figure 5.

Spectral Attributes: Program `spec_max_entropy`

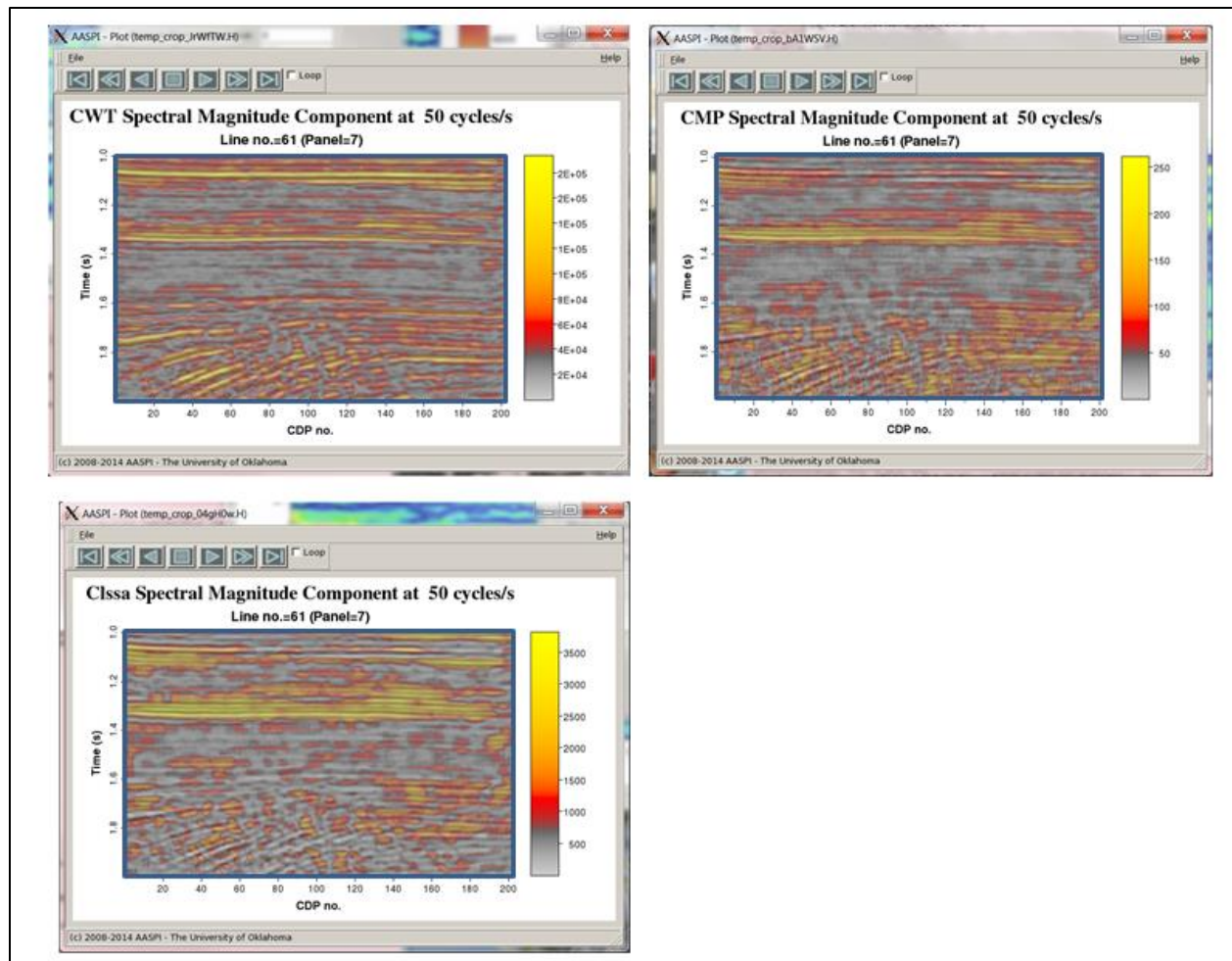


Figure 6.

Example 2: `max_entropy` applied to the Boonsville survey

Now, plot some of the results. Since we did not choose to store the spectral magnitude and phase components as a 4D cubes, we have several 3D volumes we can plot separately.

Plotting the same time slice as in all the other examples, and setting `Allpos=y` in our SEP Viewer GUI for the strictly positive magnitude, the `spec_mag_3d_max_entropy_d_20.H` (the 20 Hz magnitude component) file looks like this (see next page):

Spectral Attributes: Program **spec_max_entropy**

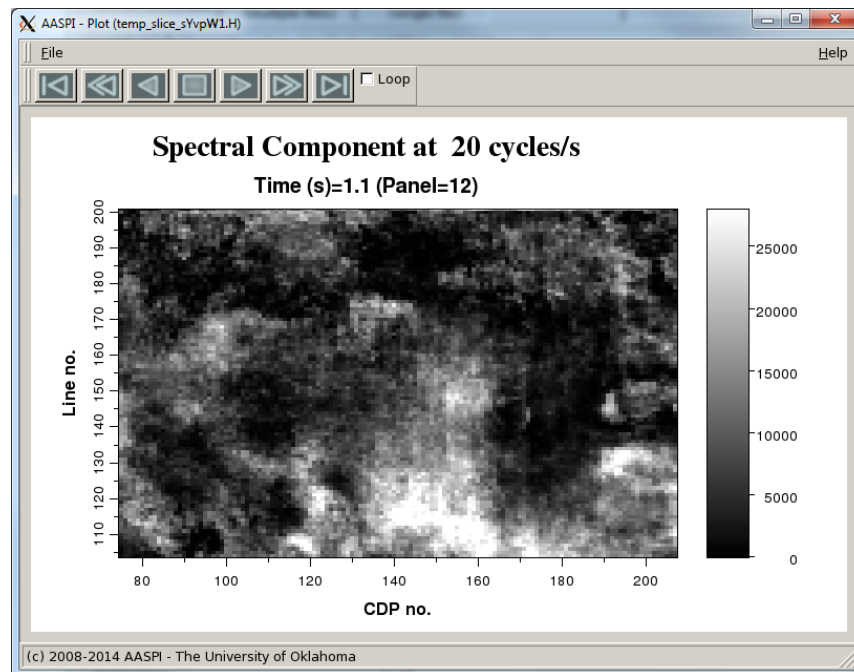


Figure 7.

While the *spec_mag_3d_max_entropy_d_50.H* (the 50 Hz magnitude component) file looks like this:

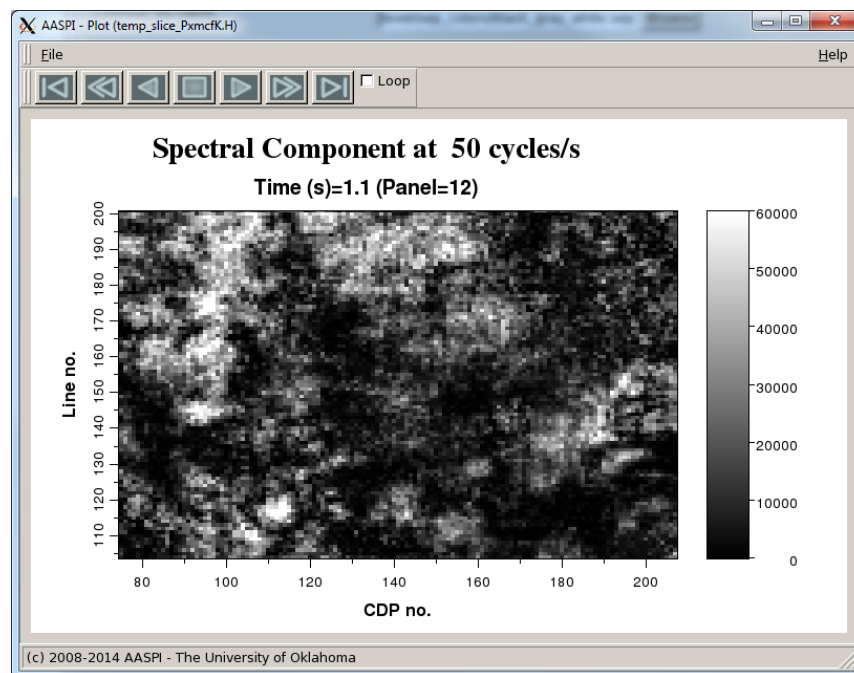


Figure 7.

Spectral Attributes: Program **spec_max_entropy**

The phase components will range from -180° to $+180^{\circ}$, so set Allpos=n and choose a cyclical color bar to plot *spec_phase_3d_max_entropy_d_20.H* (the 20 Hz phase component):

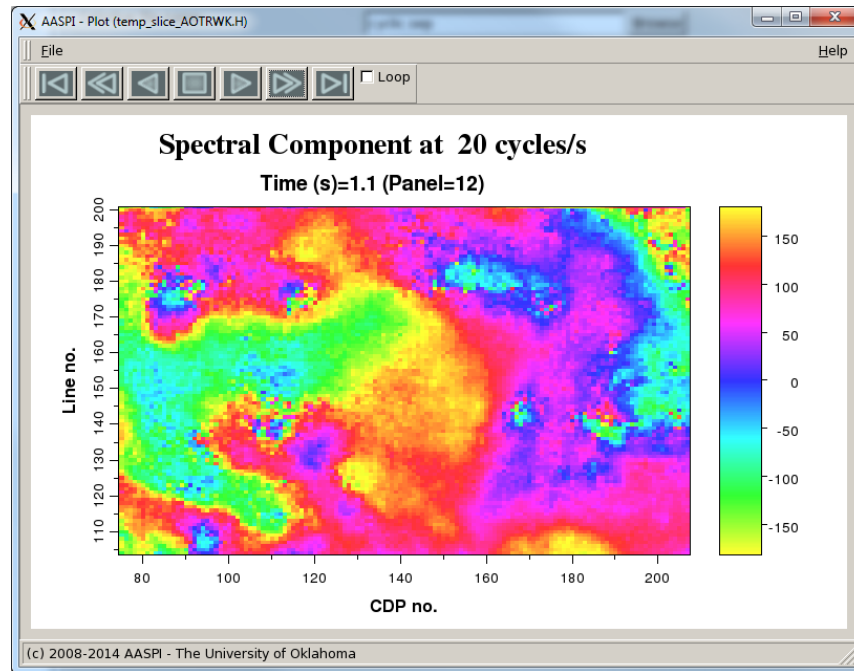


Figure 8.

and *spec_phase_3d_max_entropy_d_50.H* (the 50 Hz phase component):

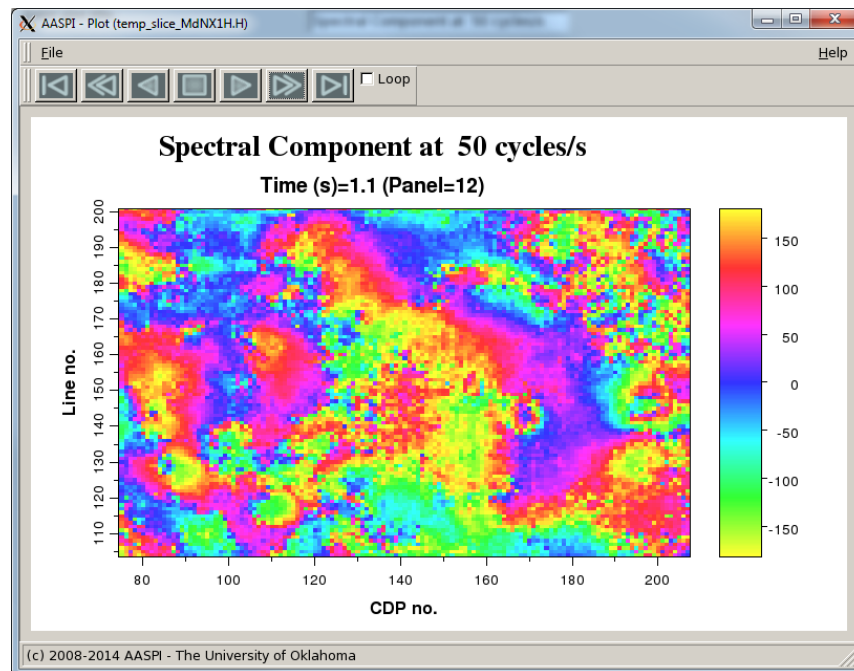


Figure 9.

Several measures of the spectrum at each time sample are also computed. The simplest one is

Spectral Attributes: Program **spec_max_entropy**

the peak magnitude (the greatest value of the spectrum) here as the file *peak_magnitude_max_entropy_d.H*:

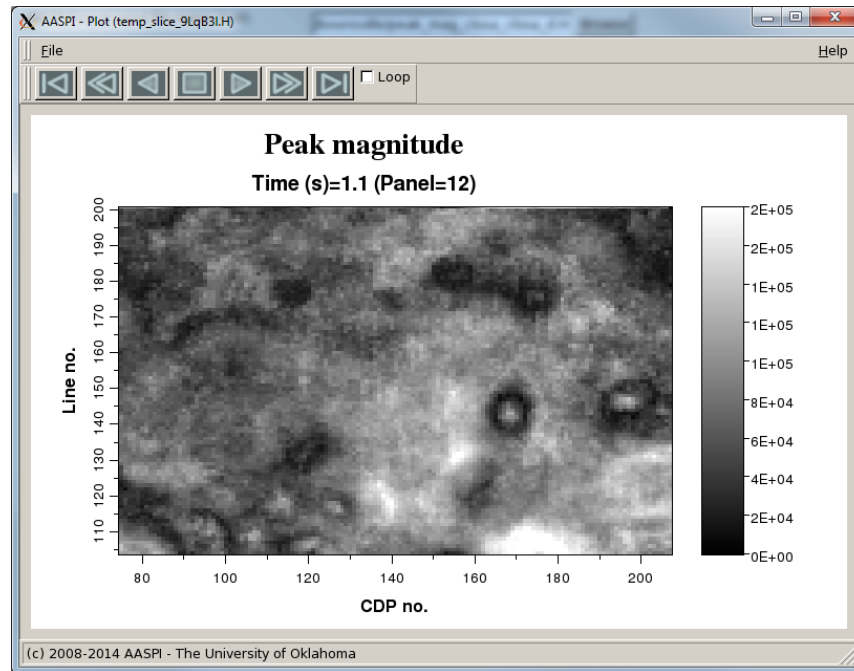


Figure 10.

We can also plot the corresponding frequency at this value (the peak frequency) which for this case is the file *peak_freq_3d_max_entropy_d.H* plotted against the *frequency.sep* (magenta-red-yellow-green_cyan_blue) color bar:

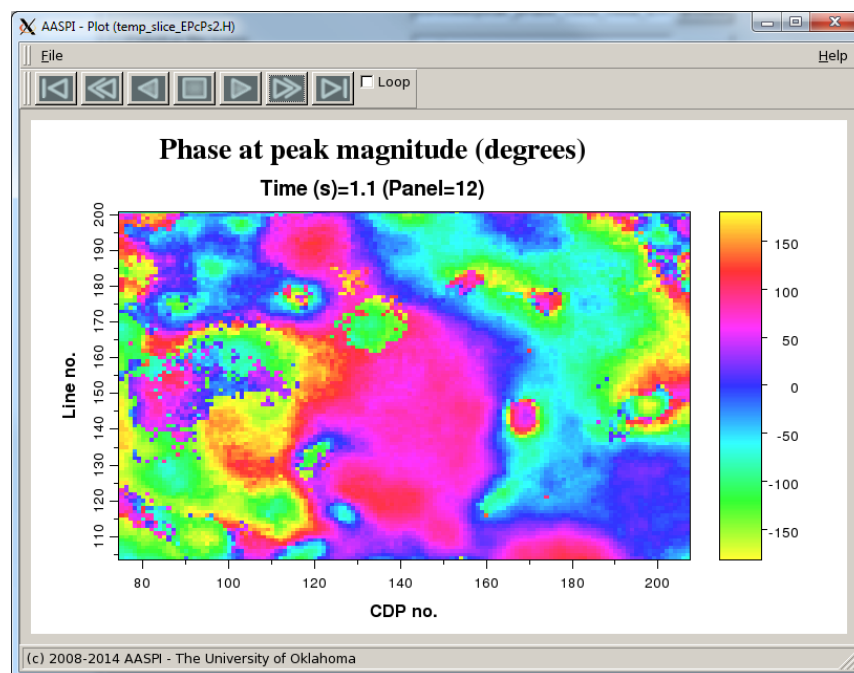
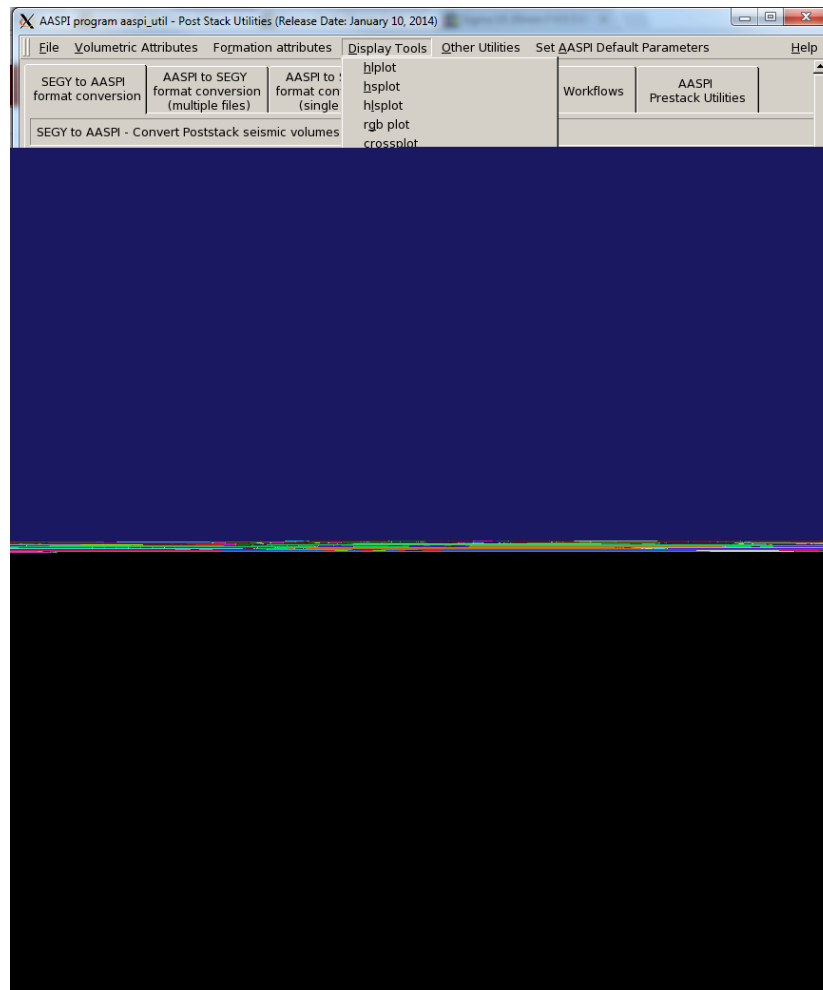


Figure 11.

Spectral Attributes: Program **spec_max_entropy**

Plotting Spectral Components

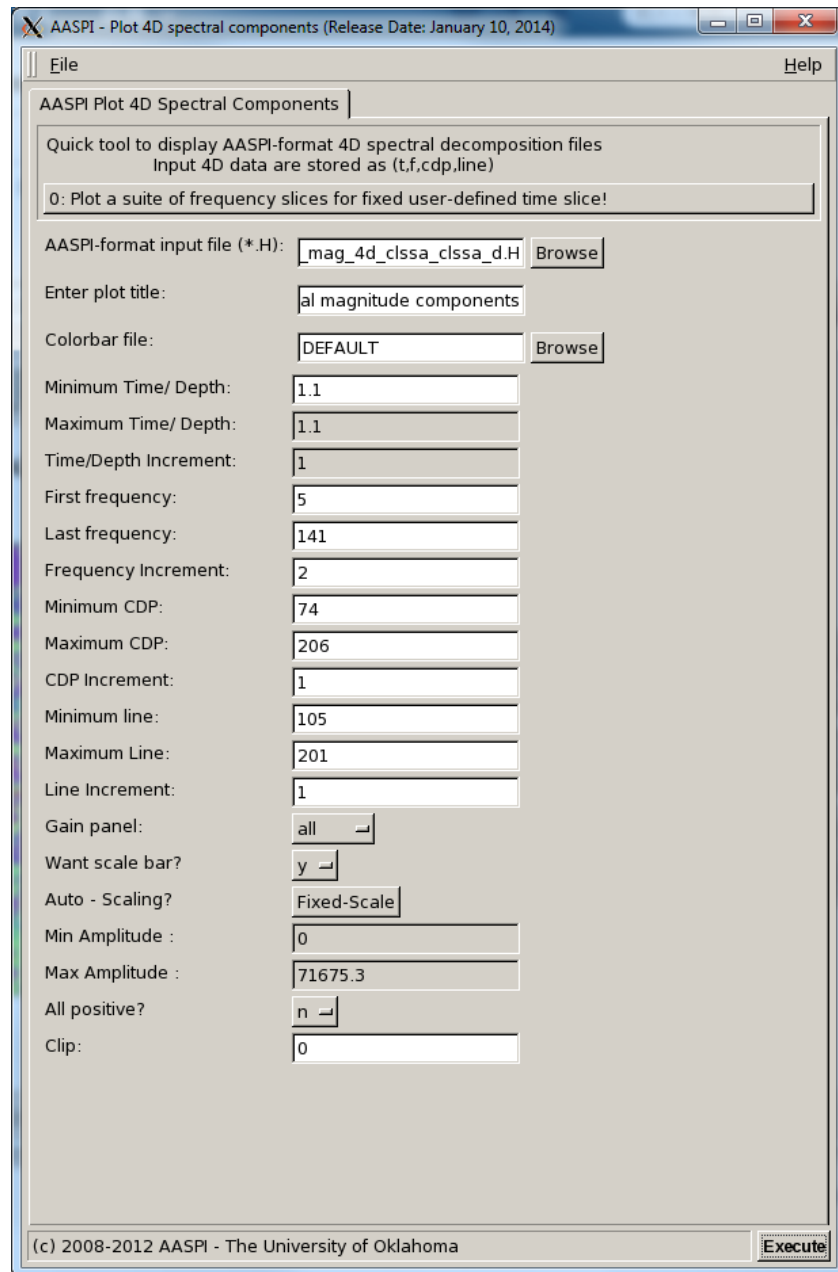
We provide a simple graphical interface to quality control the spectral components. Many commercial workstation software products now provide excellent interactive visualization of 4D volumes (t , x , y , and typically offset h , but in our case frequency, f). Our crude tool **plot_4D_spectral_components** can be found under the *Display tools* tab:



Previously, I had computed spectral components for the Boonsville survey and stored them as a 4D volume ($t, f, line_no, cdp_no$) in a file *spec_phase_4d_max_entropy_d_0.H*

Spectral Attributes: Program **spec_max_entropy**

I can also generate similar plots of the *spec_mag_4d_max_entropy_0.H* 4D volume.



I display the 30 Hz and 40 Hz spectral magnitude slices at 1.1 s:

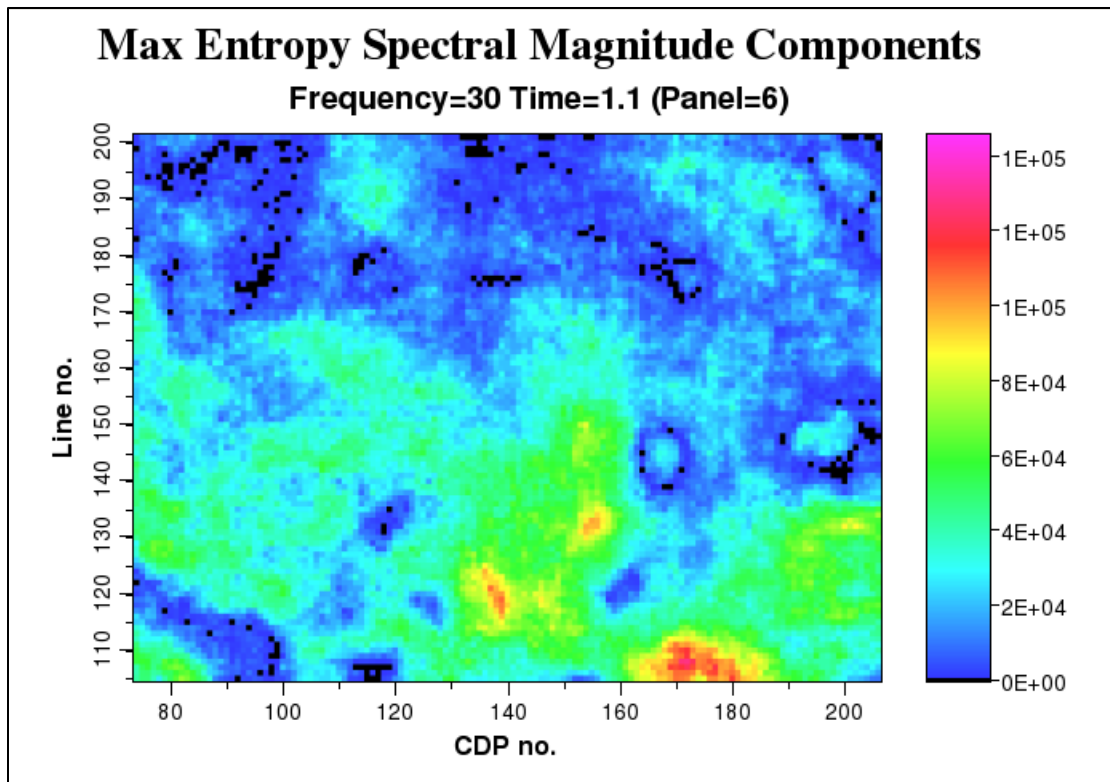


Figure 14.

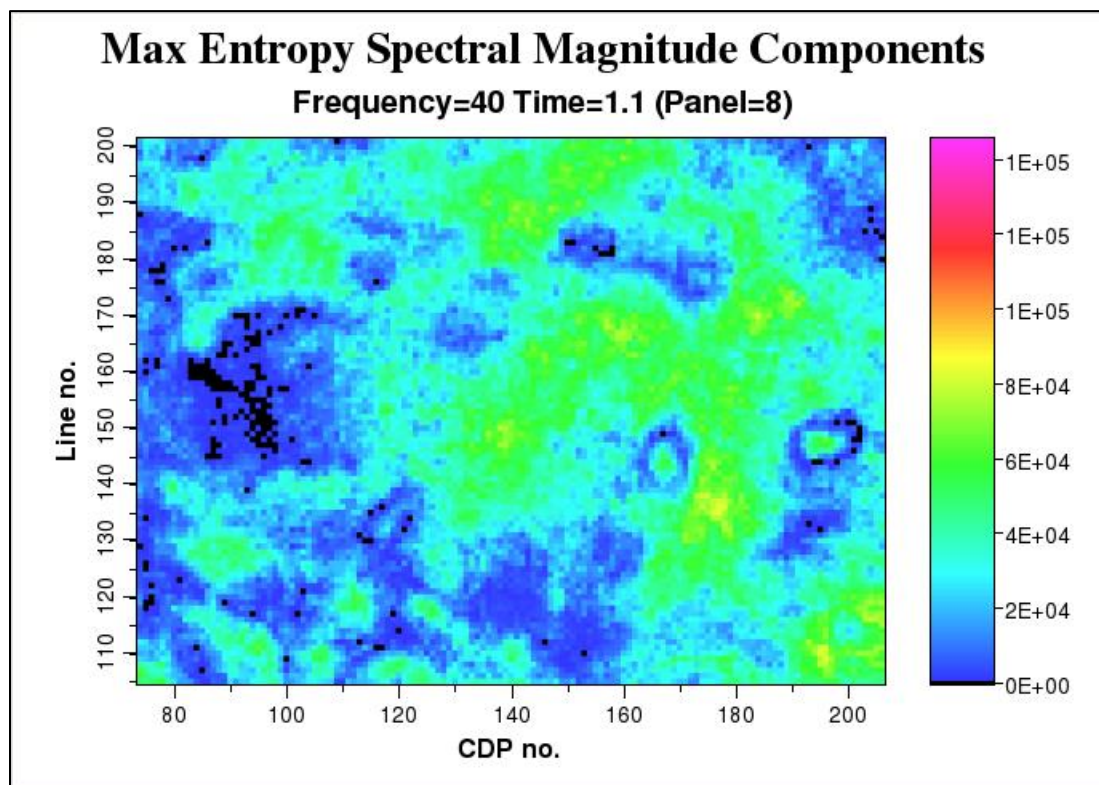


Figure 15.

References

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