Downloaded 01/23/14 to 129.15.127.245. Redistribution subject to SEG license or copyright; see Terms of Use at http://library.seg.org/

Curvature gradient attributes for improved fault characterization

Jamie Rich and Kurt Marfurt, University of Oklahoma*

Summary

Curvature gradient is a third order surface property which has been shown to improve surface and fault characterization. Previous applications have been restricted to gradients calculated in the same direction as curvature and for cases of zero dip. We demonstrate the value that can be gained by considering the full 3D tensor for variation in curvature including computation of volumetric curvature gradient in arbitrary directions relative to the curvature directions.

Introduction

Geometrical attributes are frequently used for characterizing faults and inferred fracturing (Rich and Ammerman, 2010). The most commonly applied of these are measurements of curvature. These attributes are usually calculated on either a surface or volumetrically using dip volumes as input (Al Dossary and Marfurt, 2003). When used for dip slip fault characterization, the interpretation of curvature attributes can become complex. In this case Gao (2013) demonstrated the value of considering the gradient of curvature, where the differential of curvature is considered in the maximum curvature direction to better characterize faulting.

Curvature is fully characterized by a second order tensor. At each point on a surface the curvature varies smoothly by azimuth. For geophysical characterization and interpretation the extreme values of curvature are usually considered. To fully characterize the gradient of curvature we must think in terms of a third order tensor where the curvature at some given azimuth is known to vary in any other arbitrary direction. We are then concerned with two directions, both the direction of the considered curvature and the direction in which we consider its variation.

Theory

In terms of the tensor of curvature, the quantities interpreters are most familiar with are the eigenvalues and eigenvectors of that tensor which give the extrema of the curvature values (Rich, 2008). It is common to write the curvature in some direction in terms of spanning vectors for a tangent plane at the local point of interrogation, $\boldsymbol{r} = \gamma \boldsymbol{S}_u + \delta \boldsymbol{S}_v$, as:

$$k_n = \frac{II(\boldsymbol{r},\boldsymbol{r})}{I(\boldsymbol{r},\boldsymbol{r})} = \frac{\gamma^2 L + 2\gamma\delta M + \delta^2 N}{\gamma^2 E + 2\gamma\delta F + \delta^2 G} \quad (1)$$

Where the extrema are found by considering the eigenvalue equation, $II\boldsymbol{r} = k_n I\boldsymbol{r}$ (Cipolla, 2000). I and II are the first and second fundamental form whose components are given by,

$$\begin{bmatrix} \boldsymbol{S}_u \cdot \boldsymbol{S}_u & \boldsymbol{S}_u \cdot \boldsymbol{S}_v \\ \boldsymbol{S}_v \cdot \boldsymbol{S}_u & \boldsymbol{S}_v \cdot \boldsymbol{S}_v \end{bmatrix} = \begin{bmatrix} E & F \\ F & G \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} \boldsymbol{S}_{uu} \cdot \boldsymbol{N} & \boldsymbol{S}_{uv} \cdot \boldsymbol{N} \\ \boldsymbol{S}_{vu} \cdot \boldsymbol{N} & \boldsymbol{S}_{vv} \cdot \boldsymbol{N} \end{bmatrix} = \begin{bmatrix} L & M \\ M & N \end{bmatrix} \quad (3)$$

It is useful to remind the reader that the first fundamental form gives the metric for a surface which defines both length and in general the inner product of two vectors on the surface.

In order to consider the gradient of the curvature for the case of zero dip we could consider the directional derivative,

$$\nabla_{\boldsymbol{r}} k_n = \nabla k_n \cdot \frac{\boldsymbol{r}}{||\boldsymbol{r}||} \quad (4)$$

which leads to the expression,

$$\nabla_{\boldsymbol{r}} k_n = \frac{\gamma^3 L_u + \gamma^2 \delta (2M_u + L_v) + \gamma \delta^2 (2M_v + N_u) + \delta^3 N_v}{(\gamma^2 E + 2\gamma\delta F + \delta^2 G)^{3/2}} \quad (5)$$

where,

$$\begin{aligned} L_u &= \boldsymbol{S}_{uuu} \cdot \boldsymbol{N} + \boldsymbol{S}_{uu} \cdot \boldsymbol{N}_u \\ N_u &= \boldsymbol{S}_{uvv} \cdot \boldsymbol{N} + \boldsymbol{S}_{vv} \cdot \boldsymbol{N}_u \\ L_v &= \boldsymbol{S}_{uuv} \cdot \boldsymbol{N} + \boldsymbol{S}_{uu} \cdot \boldsymbol{N}_v \\ N_v &= \boldsymbol{S}_{vvv} \cdot \boldsymbol{N} + \boldsymbol{S}_{vv} \cdot \boldsymbol{N}_v \end{aligned}$$

This is equivalent to the approach given by Gao (2013). It is immediately apparent that there are two complicating factors that introduce potential for additional value. First we may wish to consider the gradient of curvature in a direction other than the direction in which it is measured.

© 2013 SEG

DOI http://dx.doi.org/10.1190/segam2013-1290.1

Downloaded 01/23/14 to 129.15.127.245. Redistribution subject to SEG license or copyright; see Terms of Use at http://library.seg.org/

For instance it may be very useful to quantify how the maximum curvature (perpendicular to the fault) changes along the trace of the fault (minimum curvature direction). The other consideration is for the real case where dips are non-zero. In this case of non-zero dip for a surface manifold embedded in 3D space it becomes necessary to consider the covariant derivative. Replacing the directional derivative operator with the covariant derivative the expression becomes,

$$\nabla_{\boldsymbol{r}} k_n = \frac{\gamma^3 P + 3\gamma^2\delta Q + 3\gamma\delta^2 S + \delta^3 T}{(\gamma^2 E + 2\gamma\delta F + \delta^2 G)^{3/2}} \qquad (6)$$

With the terms introduced by Mehlum and Tarrou (1998),

$$\begin{aligned} P &= \boldsymbol{S}_{uuu}\cdot\boldsymbol{N} + 3\boldsymbol{S}_{uu}\cdot\boldsymbol{N}_u \\ Q &= \boldsymbol{S}_{uuv}\cdot\boldsymbol{N} + 2\boldsymbol{S}_{uv}\cdot\boldsymbol{N}_u + \boldsymbol{S}_{uu}\cdot\boldsymbol{N}_v \\ S &= \boldsymbol{S}_{uvv}\cdot\boldsymbol{N} + 2\boldsymbol{S}_{uv}\cdot\boldsymbol{N}_v + \boldsymbol{S}_{vv}\cdot\boldsymbol{N}_u \\ T &= \boldsymbol{S}_{vvv}\cdot\boldsymbol{N} + 3\boldsymbol{S}_{vv}\cdot\boldsymbol{N}_v \end{aligned}$$

Note that, in general, even if the third derivative is zero the gradient of curvature is not necessarily zero. This is important to recognize because in the case of a quadratic surface commonly considered for calculation of curvature there is still clearly a variation in curvature.

For arbitrary angles with curvature measured in direction theta and the gradient in direction phi we find a slightly more complex expression which reduces to the previous expression for the case of $\theta = \phi$.

.

$$\nabla_{\boldsymbol{r}_\phi} k_{n_\theta} = \frac{\gamma_\phi\gamma_\theta^2 P + (2\gamma_\phi\gamma_\theta\delta_\theta + \gamma_\theta^2\delta_\phi)Q + (2\gamma_\theta\delta_\theta\delta_\phi + \gamma_\phi\delta_\theta^2)S + \delta_\theta^2\delta_\phi T}{(\gamma_\theta^2 E + 2\gamma_\theta\delta_\theta F + \delta_\theta^2 G)\left(\gamma_\phi^2 E + 2\gamma_\phi\delta_\phi F + \delta_\phi^2 G\right)^{1/2}} \qquad (7)$$

The last step is to represent the components of the vector in the tangent plane in terms of the azimuthal angles. In the local $\hat{i}, \hat{j}$ coordinate plane, $\tan\theta_{az} = \delta/\gamma$. Rewriting the expression for curvature and curvature gradient in terms of $\tan\theta_{az} = \zeta$ we have,

$$k_n = \frac{L + 2\zeta M + \zeta^2 N}{E + 2\zeta F + \zeta^2 G} \qquad (8)$$

$$\nabla_{\boldsymbol{r}} k_n = \frac{P + 3\zeta Q + 3\zeta^2 S + \zeta^3 T}{(E + 2\zeta F + \zeta^2 G)^{3/2}} \qquad (9)$$

$$\nabla_{\boldsymbol{r}_\phi} k_{n_\theta} = \frac{P + (2\zeta_\theta + \zeta_\phi)Q + (2\zeta_\theta\zeta_\phi + \zeta_\theta^2)S + \zeta_\theta^2\zeta_\phi T}{(E + 2\zeta_\theta F + \zeta_\theta^2 G)\left(E + 2\zeta_\phi F + \zeta_\phi^2 G\right)^{1/2}} \qquad (10)$$

For completeness we recognize that it may also be useful to consider curvature and curvature gradient as measured in the tangent plane. In this case we consider the inner product, $\boldsymbol{S}_u \cdot \boldsymbol{r} = \|\boldsymbol{S}_u\|\|\boldsymbol{r}\|\cos\theta$. Computing this inner product using the first fundamental form the expression becomes

$$\frac{\gamma E + \delta F}{\sqrt{\gamma^2 E + 2\gamma\delta F + \delta^2 G}} = \sqrt{E}\cos\theta \qquad (11)$$

Introducing a term $\epsilon = \frac{\gamma}{\delta}$ we arrive at the solution,

$$\epsilon = \frac{1}{\zeta} = \frac{-F \pm \cot\theta\sqrt{EG - F^2}}{E} \qquad (12)$$

Volumetric curvature gradient

Al Dossary and Marfurt (2003) introduced the process for calculation of volumetric curvature where the coefficients of a quadratic surface are written in terms of volumetric dip estimates and their differentials. While it is understood that variation in curvature is non-zero for a quadratic surface, we will consider a cubic surface both for completeness and with the recognition that a cubic expression is necessary to fully characterize inflection of fault planes. Keeping the standard coefficients for a quadratic we have,

$$f(x,y) = gx^3 + hy^3 + ix^2y + jxy^2 + ax^2 + by^2 + cxy + dx + ey + f \qquad (13)$$

The coefficients given by Al Dossary and Marfurt (2003), where p and q are the inline and crossline dips respectively, are

$$\begin{aligned} a &= .5D_x p \\ b &= .5D_y q \end{aligned}$$

DOI http://dx.doi.org/10.1190/segam2013-1290.1

Downloaded 01/23/14 to 129.15.127.245. Redistribution subject to SEG license or copyright; see Terms of Use at http://library.seg.org/

$$c = .5(D_x q + D_y p)$$
$$d = p$$
$$e = q$$

The newly introduced cubic terms are,

$$g = \frac{1}{6} D_{xx} p$$
$$h = \frac{1}{6} D_{yy} q$$
$$i = .25(D_{xx} q + D_{xy} p)$$
$$j = .25(D_{yy} p + D_{xy} q)$$

Solving for the curvature coefficients in terms of the cubic coefficients we have for the first and second fundamental forms (Rich 2008),

$$E = 1 + d^2$$
$$F = de$$
$$G = 1 + e^2$$
$$L = \frac{2a}{\sqrt{(d^2 + e^2 + 1)}}$$
$$M = \frac{c}{\sqrt{(d^2 + e^2 + 1)}}$$
$$N = \frac{2b}{\sqrt{(d^2 + e^2 + 1)}}$$

and for the curvature gradient terms we have,

$$P = \frac{g}{\sqrt{(d^2 + e^2 + 1)}} - \frac{12a^2 d}{\left(d^2 + e^2 + 1\right)^{3/2}}$$
$$Q = \frac{i}{\sqrt{(d^2 + e^2 + 1)}} - \frac{4acd + 4abe}{\left(d^2 + e^2 + 1\right)^{3/2}}$$
$$S = \frac{j}{\sqrt{(d^2 + e^2 + 1)}} - \frac{4bce + 4abd}{\left(d^2 + e^2 + 1\right)^{3/2}}$$
$$T = \frac{h}{\sqrt{(d^2 + e^2 + 1)}} - \frac{12b^2 e}{\left(d^2 + e^2 + 1\right)^{3/2}}$$

It is now possible to completely characterize the curvature and curvature gradient in general volumetric terms. Curvature gradient can be computed in the same direction as the curvature or at arbitrary directions to the curvatures. The pre-computation of the curvature and gradient coefficients also allows for real-time calculation and display of the attributes for any arbitrary angles.

Interpretation

Gao et al (2013) demonstrated the benefit of gradient of curvature in characterizing faults in the Teapot Dome 3D. For comparison we use the complete tensor solution for the Tensleep horizon in the Teapot Dome 3D. Figure 1 is the maximum curvature of the Tensleep. The linear ridges bounding the faults are clearly evident on the maximum curvature display, however the actual location of the faults is not. With the curvature gradient we expect to see a negative, zero, positive anomaly at the down thrown side, followed by a zero at the axis of the fault and then positive, zero, negative sequence delineating the up thrown side. This pattern is clearly apparent on Figure 2. A fault with opposite offset would exhibit the reverse of this pattern.

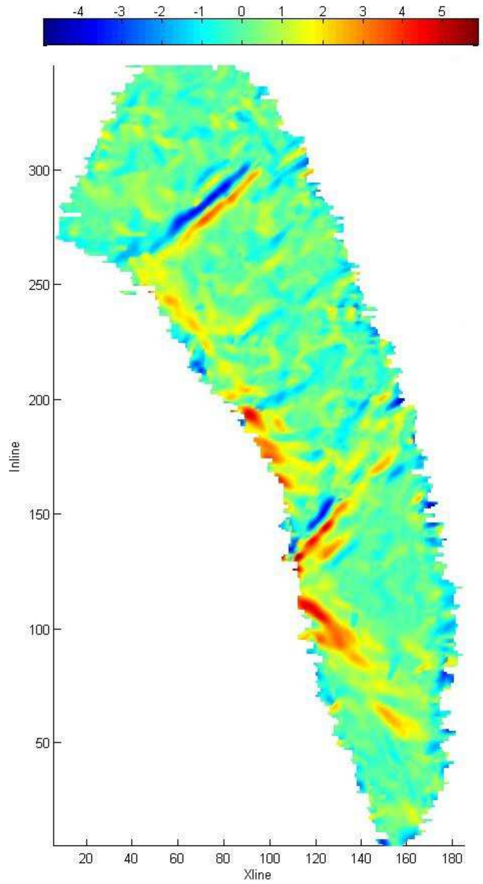

Figure 1. Maximum curvature on the Tensleep Horizon

DOI http://dx.doi.org/10.1190/segam2013-1290.1

Downloaded 01/23/14 to 129.15.127.245. Redistribution subject to SEG license or copyright; see Terms of Use at http://library.seg.org/

Not only is relative motion of the fault apparent on the curvature gradient, but the axis of the fault is better defined than on a curvature display. The reason for the pattern is an increase in negative curvature when approaching the axis on the down thrown side, followed by a decrease in negative curvature (which is a positive gradient) when moving onto the plane of the fault. The inflection point on the fault is a zero. This pattern is then reversed when approaching the up thrown side. An increase in positive curvature is a positive gradient followed by a decrease in positive curvature which is a negative gradient. Figure 3 shows the gradient of maximum curvature measured in the minimum curvature direction. While there is more 'noise' on this display, the actual fault planes, particularly the northern fault, are very well defined.

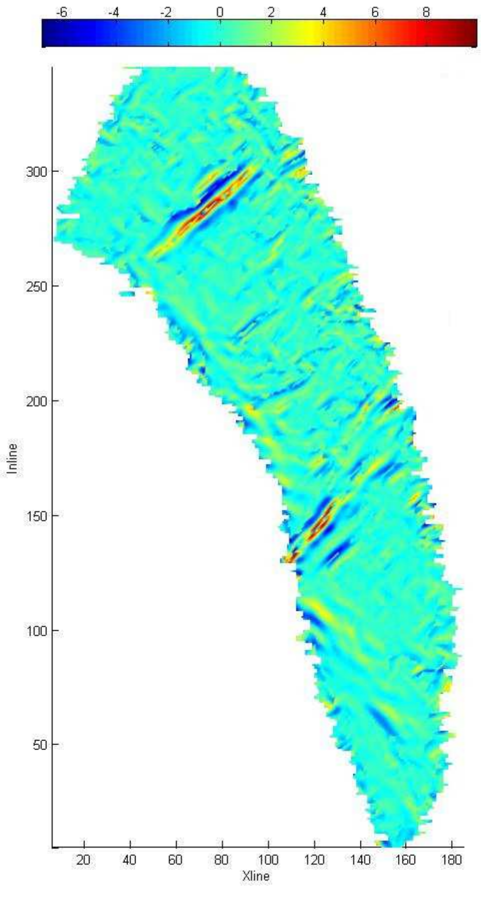

Figure 2. Gradient of maximum curvature in the direction of maximum curvature

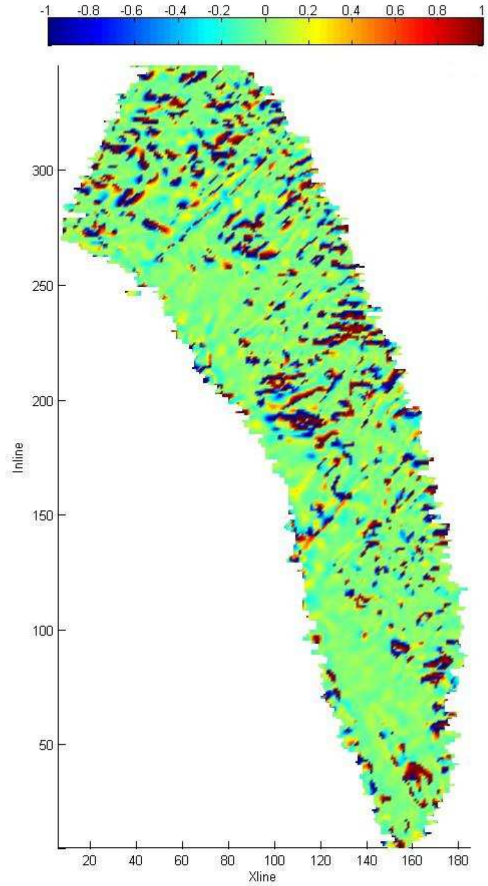

Figure 3. Gradient of maximum curvature in the direction of minimum curvature

Conclusions

Curvature is a well-known tool for characterization of both inferred fracturing and faulting. Recently curvature gradient has been introduced and shown to add value to fault characterization. We have extended the definition of curvature gradient to include both the dip terms and general azimuths of differentiation. This allows for improved fault plane characterization. In addition we have derived the expression for volumetric curvature gradient which allows for the modification of existing volumetric curvature algorithms to include general curvature gradient implementations.

Acknowledgements

We thank the RMOTC for providing the Teapot Dome 3D data.

DOI http://dx.doi.org/10.1190/segam2013-1290.1

http://dx.doi.org/10.1190/segam2013-1290.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Al-Dossary, S., and K. J. Marfurt, 2006, 3D volumetric multispectral estimates of reflector curvature and rotation: Geophysics, **71**, no. 5, P41–P51, http://dx.doi.org/10.1190/1.2242449.

Cipolla, R., and P. Giblin, 2000, Visual motion of curves and surfaces: Cambridge University Press.

Gao, D., 2013, Integrating 3D seismic curvature and curvature gradient attributes for fracture characterization: Methodologies and interpretational implications: Geophysics, **78**, no. 2, O21–O31, http://dx.doi.org/10.1190/geo2012-0190.1.

Mehlum, E., and C. Tarrou, 1998, Invariant smoothness measures for surfaces: Advances in Computational Mathematics, **8**, no. 1/2, 49–63, http://dx.doi.org/10.1023/A:1018931910836.

Rich, J. P., 2008, Expanding the applicability of curvature attributes through clarification of ambiguities in derivation and terminology: 78th Annual International Meeting, SEG, Expanded Abstracts, 884–888.

Rich, J. P., and M. Ammerman, 2010, Unconventional geophysics for unconventional plays: 2010 SPE Unconventional Gas Conference, SPE 131779.

Downloaded 01/23/14 to 129.15.127.245. Redistribution subject to SEG license or copyright; see Terms of Use at http://library.seg.org/