

## Random Noise Suppression Using Normalized Convolution Filter

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### Summary

Random noise in seismic data hampers seismic interpretation, confounds automatic pickers, overprints seismic attributes, and masks subtle geologic features of interest. For this reason much of seismic processing is devoted to increasing the signal to noise ratio. In this paper, we introduce a novel method named the normalized convolution, or NC filter, which is based on a confidence estimation of the signal, to improve our signal to noise ratio. The NC filter attenuates noise and enhances the continuity of seismic events. We demonstrate the effectiveness of the filter on simple synthetic, a real data set contaminated with real band-limited seismic noise, and a real data set contaminated with high amplitude artificial noise. These examples suggest that the proposed method is ready for application to seismic data.

### Introduction

Field seismic data almost always contain some amount of noise. Depending on its strength, spectrum, and organization, such noise can significantly impede accurate and efficient seismic interpretation. In nature, noise can be either random or coherent. Unlike coherent noise, random noise is unpredictable. Abundant methods have been proposed for seismic random attenuation. (Lu et al., 2006; Zhang et al., 2013) In general terms, these methods can be classified into global filtering and local filtering methods. Many local filtering methods have been presented and often give good results. However, most of the existing methods have a negative effect on denoising in the case of low signal-noise-ratio (SNR).

Geological structures in seismic volumes with higher SNR can be more accurately and rapidly interpreted. Autopicking and geobody extractions would work as designed. Subtle onlaps and offlaps key to seismic stratigraphic analysis can be identified and used to map sequence boundaries and infer the depositional environment. Unfortunately, strong random noise often overprints a seismic features of interest. Interpretation is tedious at best and unreliable at worst. Subtle features critical to making a play or understanding the depositional environment are easily overlooked.

Normalized convolution (NC) was first proposed by Knutsson and Westin (1993), and provides a general framework for the estimation of a local model representation of a signal (Katartzis, 2007; Westin, 1994).

NC approximates a signal with a linear combination of local basis functions. The method takes uncertainties in the signal values into account. At the same time method permits spatial localization of the basis functions which may have infinite support. Normally, NC is used as an interpolation algorithms. (Burt, 1998; Farneback, 1999)

We begin with an overview of the normalized convolution method. It is shown that one can use NC as a filter rather than as an interpolator. We then show how NC can remove random noise while enhancing the continuity of seismic events. We validate our method on synthetic and real data and find that the proposed NC filter is very effective in removing random noise and recovering important signal, especially in the cases of low SNR.

### Theory

Normalized convolution was introduced as a general method for interpolating missing and uncertain data. This method can be viewed as locally solving a weighted least squares problem. Figure 1 shows a 2D example of using NC on irregular and sparsely sampled data. Even though the sampling rate and resolution are low, we find that NC can recover most features of the original image.



Figure 1: Interpolation of a sparsely, irregularly sampled image using normalized convolution. (Left) the famous image of "Lena". (Middle) A randomly sampled image containing only 10% pixels of the original image. (Right) Reconstructed image using normalized convolution.

A general expression of linear data representation can be written as follows:

$$\mathbf{b} = \mathbf{A}\mathbf{x}, \quad (1)$$

where,  $\mathbf{b}$  is the observation, matrix  $\mathbf{A}$  is a set of basis functions, and  $\mathbf{x}$  is the input data. If  $\mathbf{A}$  is the sampling matrix,  $\mathbf{b}$  will be of lower data density than  $\mathbf{x}$ .

For a given set of basis functions  $\mathbf{A}$ , we can use weighted least squares to approximate the input data  $\mathbf{x}$ :

$$\operatorname{argmin} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_{\mathbf{w}}^2 = \operatorname{argmin} (\mathbf{b} - \mathbf{A}\mathbf{x})^* \mathbf{W} (\mathbf{b} - \mathbf{A}\mathbf{x}). \quad (2)$$

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Then, the solution  $\tilde{\mathbf{x}}$  becomes:

$$\begin{aligned}\tilde{\mathbf{x}} &= [(\mathbf{W}\mathbf{A})^T \mathbf{W}\mathbf{A}]^{-1} (\mathbf{W}\mathbf{A})^T \mathbf{W}\mathbf{b} \\ &= [\mathbf{A}^T \mathbf{W}^2 \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{W}^2 \mathbf{b}\end{aligned}\quad (3)$$

In NC, the weight matrix  $\mathbf{W}$  is divided into two parts. The first part is assigned to the basis functions and is usually referred to as the applicability function  $\mathbf{a}$ , an alternative scalar windowing function that deals with spatial localization of the operators in  $\mathbf{A}$ . The second part is assigned to the signal and is referred to as the signal certainty function,  $\mathbf{c}$ , describing the credence of the signal samples. Missing samples are handled by setting this function to zero. Note that the certainty function is usually set to zero outside the signal border, thereby reducing the impact of traditional edge effects.

Given these constructs, the diagonal weighted matrix  $\mathbf{W}$  is expressed as

$$\mathbf{W}^2 = \mathbf{W}^T \mathbf{W} = \mathbf{W}_a \mathbf{W}_c, \quad (4)$$

where,  $\mathbf{W}_a = \text{diag}(\mathbf{a})$ ,  $\mathbf{W}_c = \text{diag}(\mathbf{c})$ .

Then, equation 3 can be expressed as:

$$\begin{aligned}\tilde{\mathbf{x}} &= [\mathbf{A}^T \mathbf{W}^2 \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{W}^2 \mathbf{b} \\ &= [\mathbf{A}^T \mathbf{W}_a \mathbf{W}_c \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{W}_a \mathbf{W}_c \mathbf{b}\end{aligned}\quad (5)$$

Application of equation 5 in each local area can be efficiently implemented by means of convolution, inspiring the name normalized convolution. However, we can apply this algorithm in the opposite direction. We start by sampling the input data by ourselves, compute the confidence function, define the applicability function, and then use the two functions to estimate the noise-free data. In this way, the coherent component of the input data can be reconstructed and incoherent noise attenuated by random sampling and recovery. In order to fully reconstruct the features of the input data, we need to assure that all the input data are used through multi-sampling.

### Examples

To demonstrate the effectiveness of the method, we apply our normalized convolution filter to synthetic and real data to test its performance.

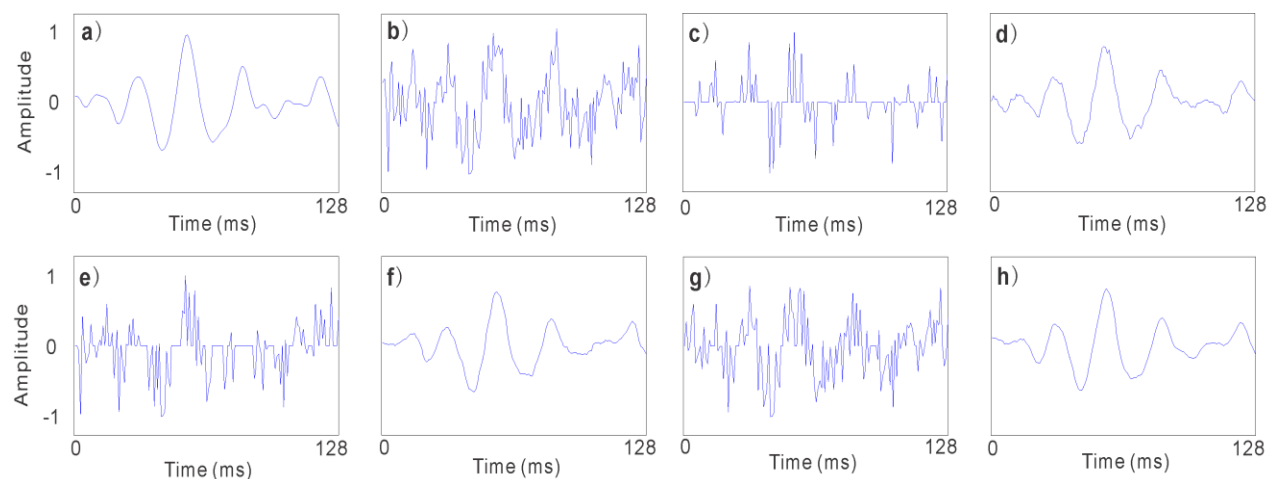


Figure 2: 1D synthetic seismic trace filtering. (a) Noise-free seismic trace. (b) 0 dB noisy trace. (c) 30% randomly sampled data. (d) Reconstructed trace from (c). (e) 50% randomly sampled data. (f) Reconstructed trace from (e). (g) 80% randomly sampled data. (h) Reconstructed trace from (g).

Figure 2 displays a 1D synthetic seismic trace filtering example. Note that the proposed method can filter most noise even at a low sampling rate; the result would be smoother at a higher sampling rate.

Figures 3-5 show the NC filter applied a noisy 2D seismic section. The input data on the left of Figure 3 is contaminated by strongly dipping coherent noise, probably due to the migration of aliased ground roll. Note on the , on

the left is filtered image to the right that random noise between seismic reflection events has been suppressed, and the continuity of the events in the red rectangles has been enhanced.

To test the capability of our approach in the cases of low SNR, we add artificial random noise to the original data in Figure 3. The input data of Figure 4 and Figure 5 are noisy with SNRs of 0 dB and -10 dB, respectively. As the definition of SNR is the common logarithm of the ratio of

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the power of signal and the power of noise, the noise in Figure 4 is as strong as the input data, while the power of noise is Figure 5 is 10 times of that of signal. Note that the filtered result of the 0 dB noise data is very similar to that

of the original data, which proves that the proposed method can remove strong noise. In addition, when the noise is 10 times stronger than the input data, we are still able to reconstruct the main features (Figure 5).

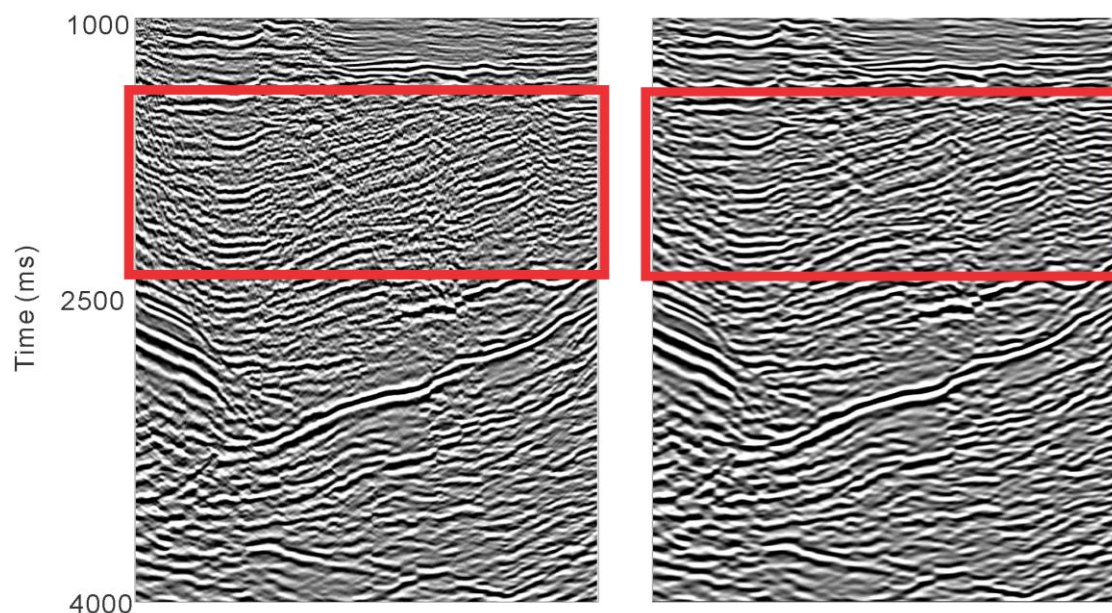


Figure 3: 2D seismic section filtering. (Left) Original seismic section. (Right) Filtered result by proposed method.

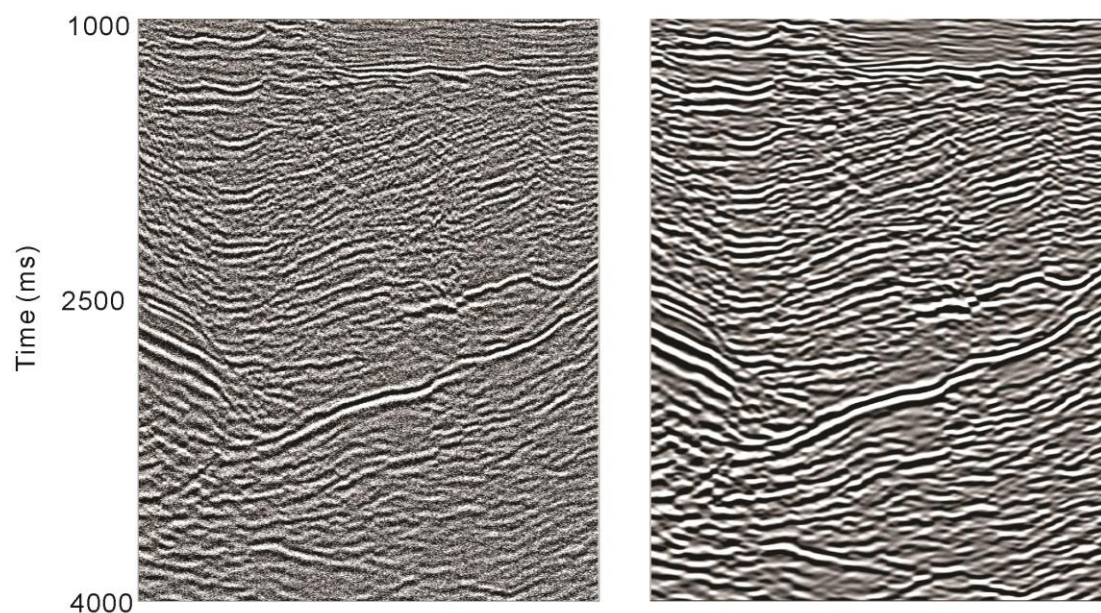


Figure 4: 2D seismic section filtering. (Left) Seismic section with 0 dB noise on original data. (Right) Filtered result by proposed method.

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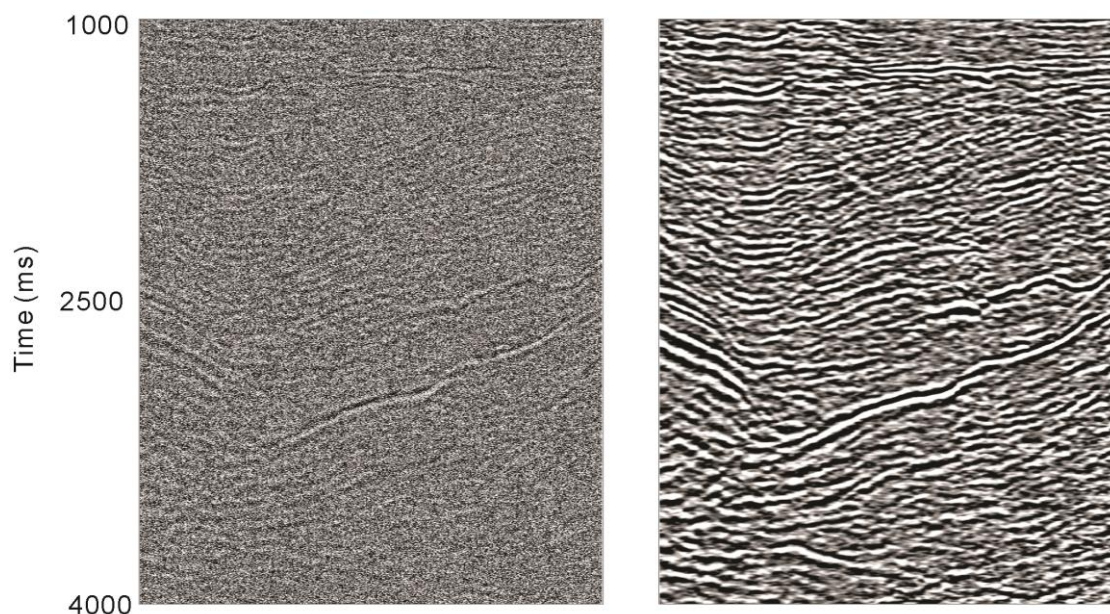


Figure 5: 2D seismic section filtering. (Left) Seismic section with -10 dB noise on original data. (Right) Filtered result by proposed method.

In our implementation of the NC filter, we set confidence function,  $c$ , to be the sampling matrix, with  $c=1$  for the sampled points and  $c=0$  for the unsampled points. The applicability function,  $a$ , used in NC defines the localization of the convolution operator. The appropriate choice of this function depends on the application. The basis function is based on the random sampling matrix given by

$$\mathbf{A} = \text{sign}(\mathbf{R} - \mu), \quad (6)$$

where,  $\mathbf{R}$  is the normal random distribution matrix,  $\mu$  is the sampling rate, and the sign function is defined as

$$\text{sign}(x) = \begin{cases} 1: & x \geq 0 \\ 0: & x < 0. \end{cases}$$

In our examples, where we added Gaussian white noise, we used a 2D Gaussian window as the applicability function:

$$a(r) = \exp\left(-\frac{|r|^2}{\sigma^2}\right), \quad (7)$$

where  $\sigma = 3$  samples laterally and 5 samples vertically and  $r$  is the distance from the sample to the point to be interpolated. We also used a Gaussian window for the real band limited noise shown in Figure 3. If the noise is non Gaussian, we should choose the applicability function suitable for the input data. In addition, the localization character is also important, if the applicability function is too local, we will recover the input data as well as the

noise; if it is wider, the filter will suppress the noise in the neighborhood area.

### Future Work

Our first implementation is “dip unaware”. The first pass of the filter as shown above may allow us to estimate more accurate structural dip. As a second pass, we wish to design confidence and applicability functions that honor the local structure information resulting in a structure-oriented filter.

### Conclusions

We develop a local filtering method based on normalized convolution to attenuate the random noise in seismic data. By testing this approach on synthetic and real data, we demonstrate that the proposed filter has a clear effect on suppressing random noise and can recover seismic events. Comparing the results of input data with different SNR, we find that our approach has wide applicability, especially in the cases of noisy seismic data with low SNR.

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#### EDITED REFERENCES

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