

#### Introduction

Partyka et al. (1999) first showed how spectral decomposition can be used to map lateral changes in acoustic properties and thickness of stratigraphic layers. Since then, spectral decomposition has become a popular tool to be applied to depositional features recognition (Liu and Marfurt, 2007), direct hydrocarbon detection (Castagna et al., 2003), and thin bed thickness determination (Marfurt and Kirlin, 2001). However, to our knowledge, most of the published applications are based on the magnitude component of the joint time-frequency spectrum, and few if any has been introduced in the spectral phase response due to stratigraphy, or on mapping spectral phase discontinuities.

Different stratigraphic packages due to changes in the depositional systems or diagenesis (in carbonates) (Hart, 2008) generate different phase changes. Although phase rotations can be introduced through seismic processing and may be further adjusted by the interpreter, some of the geology-induced phase shifts can be easily identified, such as spatial discontinuities associated with faults and incised channels, but phase shifts due to condensed sections and erosional unconformities can be quite subtle. Stark (2003) showed that high gradient values of the 2D unwrapped instantaneous phase obtained from complex trace analysis can delineate some of these features.

In this paper, we show how the phase changes as a function of the frequency and time in the spectral decomposition result, and how such changes can be related to seismic stratigraphy. We begin with a review of some theories on joint time-frequency phase unwrapping. Then, we show how to simply extract a new discontinuity attribute (phase residue, or anomalies in the phase spectra) instead of taking the Laplacian of the unwrapping phase (Matos et al., 2009). Finally, we calibrate these phase residues computed from a seismic data acquired over the Anadarko Basin, Oklahoma, USA, against well control to validate the resolution of thin sands that fall at the limit of seismic resolution.

## Joint time-frequency phase unwrapping

Matos et al. (2009) reviewed several different ways to unwrap one-dimensional seismic phase and these techniques provide similar results if the phase is not aliased or noise-free. One might anticipate that the phase unwrapping of the spectral decomposition result can be simply solved by unwrapping the phase from each frequency using the one dimensional techniques. However, unwrapping the phase using this "component independent" process will generate phases discontinuities across frequency. Likewise, unwrapping the phase along time axis will generate artifacts across time direction. This shortcoming suggests taking into account the phase relationships along the frequency as well as the time axis to unwrap the data, thereby requiring a two-dimensional phase unwrapping strategy.

Basically, two-dimensional phase unwrapping is embodied in the evaluation of the line integral:

$$\varphi(\mathbf{r}) = \int_{P} \nabla \varphi d\mathbf{r} + \varphi(r_0) \quad (1)$$

where P is any path from point  $r_0$  to r. Aliasing, singularities, and noise can make the above integration highly dependent on the integration path. We wish that equation 1 depends only on the end points of path P and not a specific path selected.

Below we will introduce two of the most promising two dimensional phase unwrapping techniques Ghiglia and Pritt (1998) introduced: quality-guide path and least-squares approaches. For the quality-guide path technique, the first step is to calculate quality map defined by Matos et al. (2009). In the second step, we take the quality map as a guide to unwrap each time-frequency surface using flood algorithm starting at a reference point where the time and frequency is set to zero.

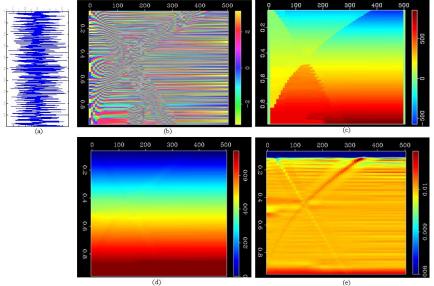
The least-squares approach for the phase unwrapping is given by solving Poisson's equation:

$$(\phi_{t+\Delta t,f} - 2\phi_{t,f} + \phi_{t-\Delta t,f}) + (\phi_{t,f+\Delta f} - 2\phi_{t,f} + \phi_{t,f-\Delta f}) = W(\psi_{t+\Delta t,f} - \psi_{t,f}) - W(\psi_{t,f} - \psi_{t-\Delta t,f}) + W(\psi_{t,f+\Delta f} - \psi_{t,f}) - W(\psi_{t,f} - \psi_{t,f-\Delta f})$$
(2)



Where  $\phi$  and  $\psi$  are unwrapped and wrapped phase respectively, W is the wrapped operator that wraps its argument value into the range  $(-\pi, +\pi]$  by adding or subtracting an integer multiple of  $2\pi$  radians to its argument. Equation 2 can be solved by Cosine transform directly in frequency domain.

Figure 1 gives the comparison of the unwrapping results based on above two approaches. Obviously, the unwrapped phase from least square approach 1d is much smoother than quality-guide path result 1c. However, the AGC of 1d shows the exact time-frequency distribution of the original signal. To obtain a better solution, the above unweighted least-squares approach can be modified to a weighted least-squares problem in which weights are generated by quality map or spectral magnitude.



**Figure 1:** The comparison of two phase unwrapping solutions with (a) signal, (b)time-frequency wrapped phase, (c) unwrapping results based on quality-guide path, (d) unwrapping results based on least-squares approach, and (e) is the AGC result of (d).

## Phase residual

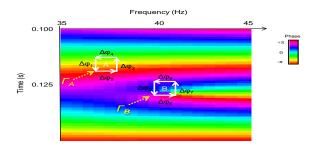
Ghiglia and Pritt (1998) show how a complex residue theorem based on vector calculus can be applied to the phase unwrapping problem. Specifically, they chose the smallest possible path, i.e., around every 2x2 sample in the 2D wrapped phase matrix and calculate the path integral as described,

$$I = \frac{W\{\psi(t + \Delta t, f) - \psi(t, f)\}}{2\pi} + \frac{W\{\psi(t + \Delta t, f + \Delta f) - \psi(t + \Delta t, f)\}}{2\pi} + \frac{W\{\psi(t, f + \Delta f) - \psi(t + \Delta t, f + \Delta f)\}}{2\pi} + \frac{W\{\psi(t, f + \Delta f) - \psi(t, f + \Delta f)\}}{2\pi}$$
(3)

If I in equation 3 is non-zero there are inconsistent points which they call phase residues. Figure 2 shows how the residue is calculated for a small portion of a typical wrapped time-frequency phase matrix. It can be easily proved that the only possible values for the phase residue are 0 and  $\pm 1$ . Workers who choose to unwrap the phase try to avoid the phase residues in some manner. Our objective is much less ambitious. Instead of unwrapping the phase, we will simply display phase residuals properties that appear in the joint time-frequency distribution as seismic attributes that can be associated with busts in either data quality or stratigraphic discontinuities.

In this paper we defined three new seismic attributes: the frequency where each phase residue occurs, the phase value at the residue location and the corresponding amplitude spectrum at this frequency. We tested these attributes on a synthetic channel seismic model in Figure 3. By plotting the maximum amplitude at the location of phase residues we can clearly see the detected channel 3b.





**Figure 2:** Computation of the phase residue. The integration  $\Gamma_A$  about point A is  $\Delta \varphi_1 + \Delta \varphi_2 + \Delta \varphi_3 + \Delta \varphi_4 = 0$  and the integration  $\Gamma_B$  about point B gives  $\Delta \varphi_5 + \Delta \varphi_6 + \Delta \varphi_7 + \Delta \varphi_8 = 1$ , giving rise to a residue.

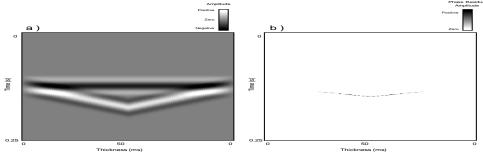


Figure 3: Channel model (a) and Phase residue amplitude attribute (b) showing the detected channel.

# Real data example

To demonstrate the value of phase residues we use a seismic data volume that served as one of the first published applications of spectral decomposition (Peyton et al., 1998). Ten years and 100s of wells later in this survey, the Red Fork channels of the Anadarko Basin of Oklahoma are still problematic. The incised valleys have undergone at least five stages of incision and fill. Suarez et al. (2008) reported that fill can be comprised of lag deposits, shales, coals, muddy sands, and sands. In addition to the seismically-resolved incised channels, Suarez et al. (2008) report some channels can be seen by the drill bit and well logs but not by seismic data. We selected 51 wells within the survey that contained gamma ray and SP curves to calibrate our phase residue anomalies.

Figure 4 shows some results from line A-A' where we co-render the seismic amplitude, the phase residues and gamma-ray logs, we recognize a correlation of the sand signatures in the GR logs with lineaments in the phase residue. The phase residue amplitude and frequency were blended using a two-dimensional colormap and allow us to interpolate these thin sands between the sparse well control. These sands are not apparent in the seismic amplitude data due to the limited vertical resolution. Line B-B' (Figure 5) shows also a similar good correlation between the phase residue and the GR log which we interpret to be a sand pinch out.

### **Conclusions**

We show how phase residues can be a geologically meaningful attribute. Through the real data calibration, we interpret these phase residues appear to be sensitive to subtle discontinuities that are not easily seen in input seismic amplitude data. We believe phase residues are sensitive to the some kinds of stratigraphic discontinuities seen by analysis of the magnitude component of time-frequency distribution described by Li and Liner (2008). Since the phase is often a more robust seismic measure than magnitude, we believe it holds significant promise in mapping stratigraphic unconformities.

## Acknowledgements

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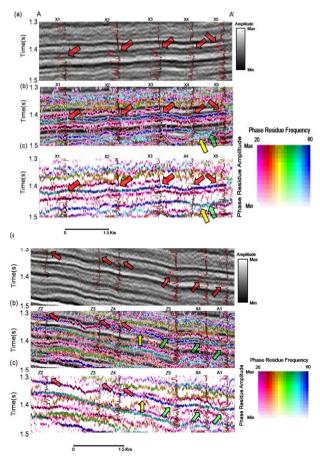


Figure 4: Vertical section A-A' trough seismic data (a), (b) seismic amplitude data co-rendered with phase attributes, and (c) phase attributes. Red arrows indicate de base of a GR sequence. Yellow arrow indicates a channel-like feature in the phase attributes that is masked in the seismic data. The green arrow is interpreted as the levee of the channel interpreted by the yellow arrow or an adjacent channel that is not resolved in the seismic section. In (c) the sand body reflection interface is better resolved in the phase residue attribute.

Figure 5: Vertical section B-B' trough seismic data (a), (b) seismic amplitude data co-rendered with time-frequency phase attributes using two dimensional colormap and (c) time-frequency phase attributes using two dimensional colormap. (a) red arrows indicate a continous sand body following the GR log response and the seismic data. (b) with the aid of the attribute the continous sand body in (a) is reinterpreted as two different sand bodies at different levels (red and green arrows). The yellow arrow is interpreted as a pinchout of the lower sand body. The lower sand body is better delienated by the time-phase residue (c).

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