Volumetric Aberrancy: Quantifying the magnitude and orientation of faults and flexures

Xuan Qi, Kurt Marfurt

What is it?

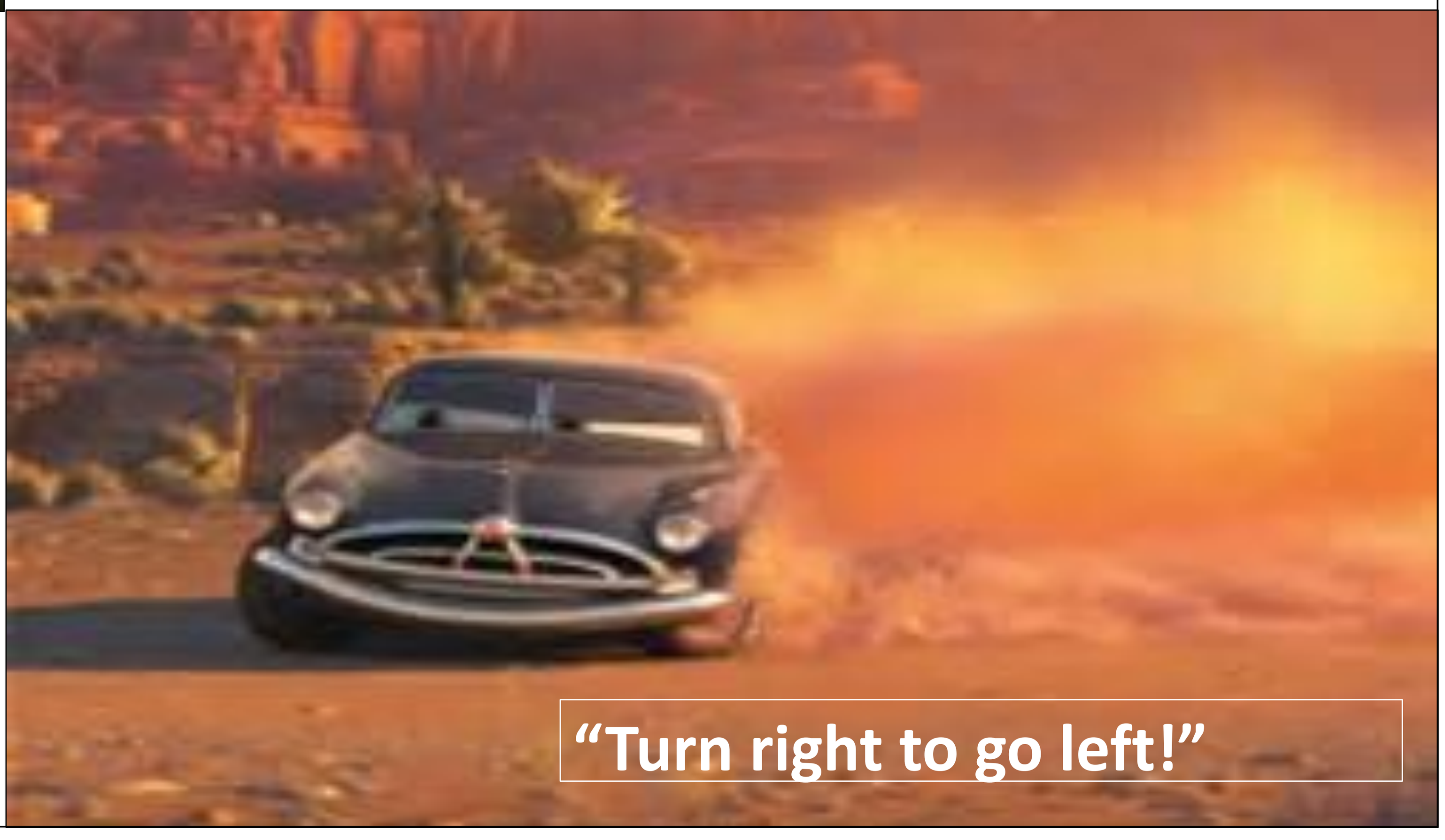
Position, *s(x)*

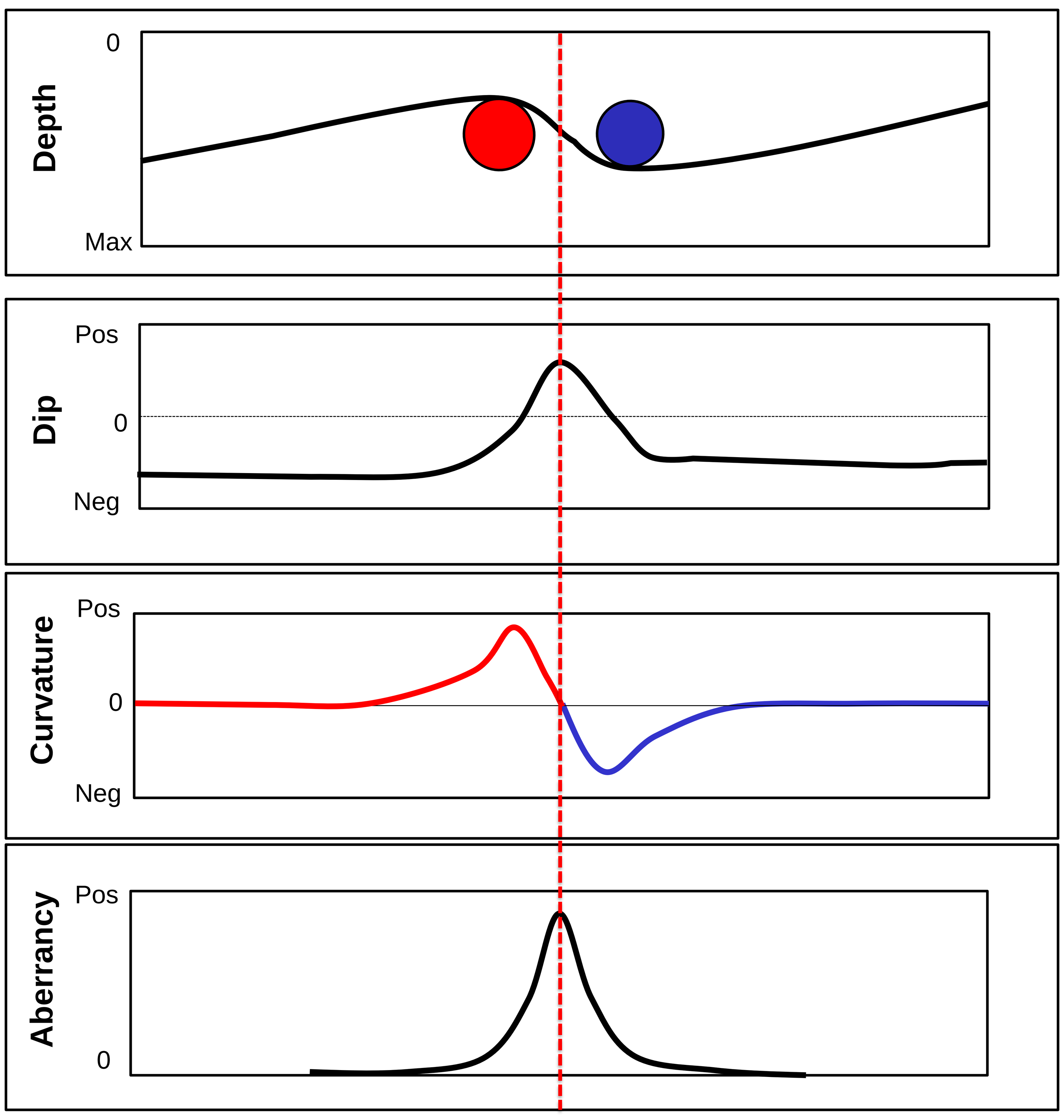
Velocity, *ds/dx*

Acceleration, d^2s/dx^2

Jerk, d^3s/dx^3

Aberrancy is the third derivative of a surface. Geoscientists learn about position, velocity, and acceleration in high school physics, and that velocity is the first derivative and acceleration the second derivative of the surface position with time in first year calculus. The third derivative is jerk. Think of the experience of a roller coaster as you are "jerked" by a rapid change in the curve of the track. If you are younger, think of "Doc" in the movie "Cars" where you are taught to turn right to go left.





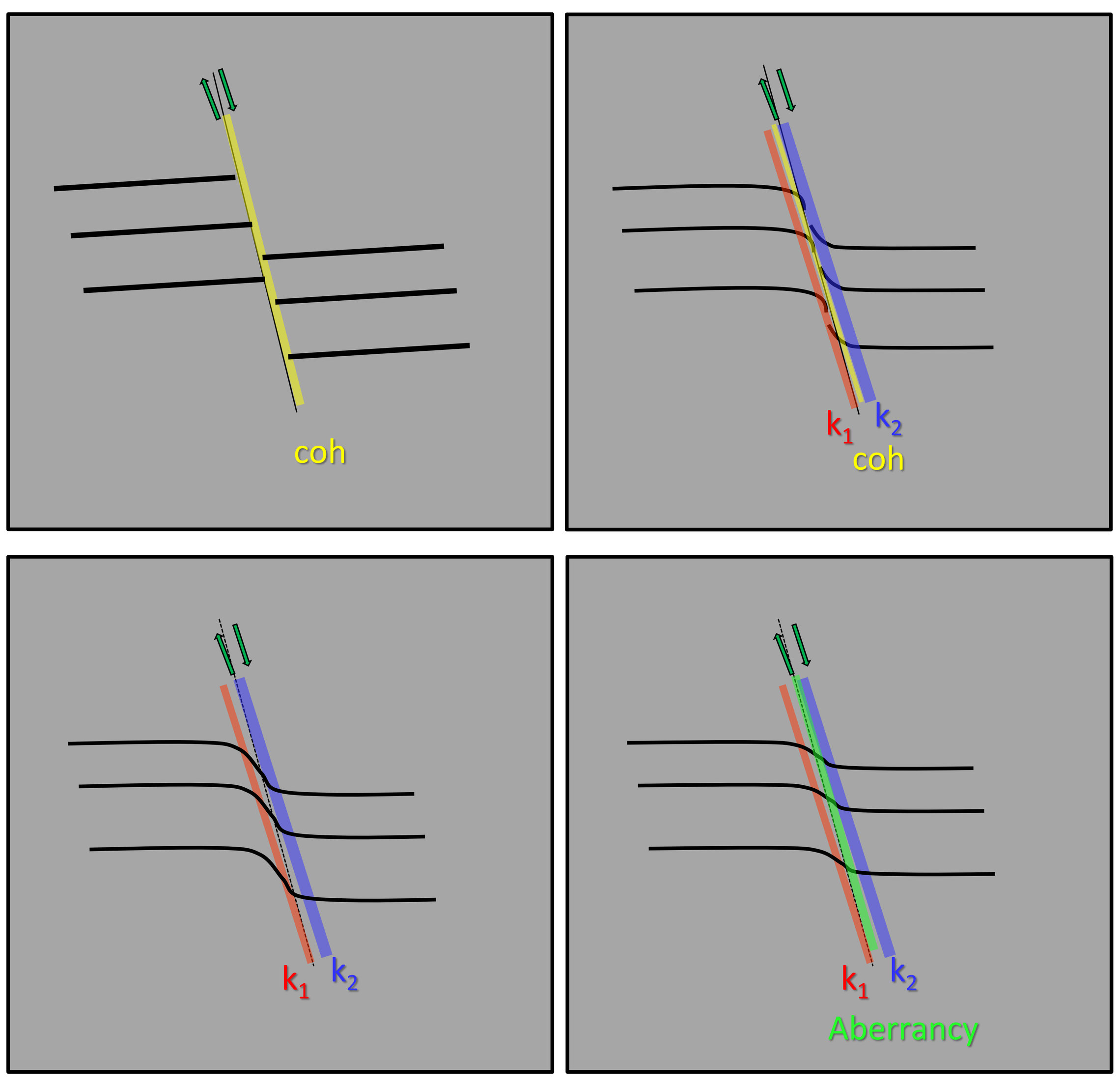
Aberrancy: the gradient or measure of lateral change in curvature

Aberrancy measures the lateral change in curvature along a surface. (a) A cartoon of a monocline and syncline. (b) The dip magnitude is the 1st derivative of structure. On the left and the right of the monocline the dips approach zero. The dip magnitude in the center is maximum while the dip azimuth is to the right. (c) Curvature is the 2nd derivative of structure. Using the previous definition, the anticlinal half of the monocline exhibits positive curvature while the synclinal half exhibits negative curvature. The curvature in the center and towards both sides of the model are zero. (d) Aberrancy is the 3rd derivative of structure. The aberrancy magnitude will be maximum in the center of the image where the flexure is greatest and decreases towards the edges. The aberrancy azimuth is towards the right.

Why do we care?

Fault offset only, well imaged

Fault offset with conjugate faulting or poor imaging



Fault offset (if any) below resolution

Fault offset (if any) below resolution

Seismic attributes are routinely used to highlight faults. Different attributes highlight different features of the fault system, only some of which are shown here. For instance, (a) coherence illuminates faults that have finite offset, such that the waveform does not match across the fault. The small displacement along multiple conjugate faults adjacent to the larger main fault often gives rise to curvature anomalies (b) – a positive curvature anomaly on the footwall, and a negative curvature anomaly on the hanging wall. The conjugate faults themselves fall below seismic resolution so that feature looks like a flexure. The curvature anomalies bracket the coherence anomaly. If the conjugate faults take up most of the displacement (c), the coherence anomaly disappears. However, we can still "track" the fault by the two bracketing curvature anomalies. (d) A more recent development, aberrancy (sometimes called flexure) measures the change in curvature. In this example, the curvature changes most rapidly about the main fault location.

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What are some of the mathematical details?

Internal Steps

Step 1: Compute Derivatives

Step 2: Rotate Coordinate system

Step 3: Compute the extreme aberrancy

Step 4: Rotate back to original coordinate system

Aberrancy is based on the 3rd derivatives of a explicitly or implicitly defined surface, which is equivalent to the 2nd derivatives of the vector dip components. In this latter case, the surfaces are implied implicitly, facilitating a volumetric computation. Di and Gao's (2014) computation was a computationally intensive numerical search over azimuths for the maximum and minimum values of aberrancy. In 2016, Di and Gao showed how the equations become more tractable by first flatting the coordinate system about the dip at each surface analysis point. This latter approach works well for horizons but because of our three-dimensional convolutional implementation of the derivatives becomes computationally too intensive for volumetric analysis. Instead, we first compute the 2nd derivatives in the original coordinate system. This 3x3 derivative operator is then rotated at each voxel to form the necessary components for the simplified aberrancy equations.

Step 1: computation of derivatives

1st derivative in Fourier domain

Low pass filter in wavenumber, k

Operator δr defined along axis r

2nd derivatives in the rotated coordinate system

Rotational matrix, R

$$\delta r \approx F^{-1}\left[-ikA(k)\right] \longrightarrow \frac{\partial u}{\partial r} \approx \delta r * u \longrightarrow \frac{\partial^2 p'_k}{\partial x'_i \partial x'_j} = \sum_{l=1}^{3}\sum_{m=1}^{3}\sum_{k=1}^{3} R_{im}R_{jl}R_{kn}\frac{\partial^2 p_n}{\partial x_m \partial x_l}$$

All derivatives are first computed in the unrotated (world) coordinate system using a "long-wavelength" operator, δr. Next, rotate each derivative operator, $\frac{\partial u}{\partial r}$, as well. Which, when combined, gives the 2nd derivatives in the rotated coordinate system in terms of the 2nd derivatives in the original (unrotated coordinate system).

Step 2: aberrancy after rotation

2nd derivatives of dip in rotated coordinate system

2nd derivatives of dip in rotated coordinate system

2nd derivatives of dip in rotated coordinate system

2nd derivatives of dip in rotated coordinate system

$$f(\psi) = \frac{\partial^3 z'}{\partial x'^3}\cos^3\psi + 3\frac{\partial^3 z'}{\partial x'^2 \partial y'}\cos^2\psi\sin\psi + 3\frac{\partial^3 z'}{\partial x' \partial y'^2}\cos\psi\sin^2\psi + \frac{\partial^3 z'}{\partial y'^3}\sin^3\psi$$

aberrancy

ψ: azimuth [-180°,+180°]

Step 3: find the roots tanψ of

$$\frac{df(\psi)}{d\psi} = 3\cos^3\phi_1 \cdot \left[-\frac{\partial^3 z'}{\partial x' \partial y'^2}\tan^3\psi - \left(2\frac{\partial^3 z'}{\partial x'^2 \partial y'} - \frac{\partial^3 z'}{\partial y'^3}\right)\tan^2\psi + \left(2\frac{\partial^3 z'}{\partial x' \partial y'^2} - \frac{\partial^3 z'}{\partial x'^3}\right)\tan\psi + \frac{\partial^3 z'}{\partial x'^2 \partial y'}\right] = 0$$

ψ: azimuth [-180°,+180°]

Conclusion

Tectonic forces, diagenetic dissolution, diapirism, and erosion all act to deform stratigraphic layers that originally may have been deposited with relatively featureless surfaces. While coherence measures disruptions in these surfaces, dip, curvature, and aberrancy measures changes in their orientation and morphology. Lateral changes in dip give use to curvature while lateral changes in curvature give use to aberrancy.

Previously limited to computation from picked horizons, we have extended aberrancy to provide volumetric results of uninterpreted seismic data volumes. By using along wavelength calculations commonly used in volumetric curvature computations implemented as convolution operators in the original unrotated data volume, we obtain results that are numerically stable, computationally efficient, and geologically meaningful.

While aberrancy will provide superior images of certain geologic features, it will complement rather than supplant other structural attributes such as coherence, curvature, and diffraction imaging. Indeed, when used together, they provide deeper insight into the seismic data volume.

Reference

- Gao, D., and H. Di, 2015, Extreme curvature and extreme flexure analysis for fracture characterization from 3D seismic data: New analytical algorithms and geologic implications: Geophysics, **80**. IM11-IM20, doi:10.1190/geo2014-0185.1.
- Marfurt, K. J., 2006, Robust estimates of 3D reflector dip and azimuth: Geophysics, 71, P29–P40, doi:10.1190/1.2213049.

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