The synchrosqueezing generalized S-transform algorithm: Application to the Sichuan Basin

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Summary

A new time-frequency analysis method—Synchrosqueezing Generalized S-transform (SSGST)—is proposed to meet the needs of high-resolution seismic signal processing and interpretation. The basic wavelet of the generalized S-transform (GST) is a modulated harmonic wave with four undetermined parameters that can be constructed by adjusting the four parameters to make the GST more suitable for seismic signals processing. The SSGST method squeezes and reconstructs the complex coefficient spectra of GST results along the frequency direction so that the energy distributions on the time-frequency spectra are concentrated around the real instantaneous frequency of the signal; thus, the time-frequency resolution can be improved. Based on mathematical theory, we strictly deduced the mathematical expressions of the positive transformation and lossless inverse transformation of SSGST. The experimental results of numerical signals illustrate that the proposed method can correctly decompose signals with different spectral characteristics into a high time-frequency resolution spectrum and can recovery the original signal from the time-frequency spectrum with satisfying reconstructing accuracy. Application on field seismic data shows the superiority of the new method in seismic time-frequency analysis for hydrocarbon detection.

Introduction

Time-frequency analysis, a powerful tool for seismic data analysis, plays a significant role in oil and gas exploration and development. It maps a 1D signal in the time domain into a 2D time-frequency spectrum, which can effectively reveal important details of seismic data and provide valuable information for reservoir characterization.

Synchrosqueezing transform (SST) is a relatively new technique based on the combination of time-frequency methods followed by a reassignment step (Auger, 2013; Tary, 2017) method. SST can improve the energy concentration of time-frequency representation by applying a post-processing reallocation to the original representation. At present, many efforts have been made to use this method for seismic data processing and interpretation.

Herrera et al. (2014) identified channels from seismic data using SST successfully. In 2015, they also used SST to separate the P and S waves of microseismic data. Mousavi et al. (2016) used it for microseismic detection. Mousavi and Langston (2017) showed that SST can be used to improve the denoising of seismic data. Tary et al. (2017) used it for attenuation estimation.

The generalized S-transform (GST) introduced by Gao et al. (2003) overcomes the dilemma of the fixed wavelet in ST by introducing four undetermined parameters (amplitude, energy decay rate, energy delay time, and video rate) to construct the basic wavelet adaptively to the non-stationary signal characteristics in practical application. Due to having no restriction of the time window length, GST can obtain real time-frequency spectra with excellent time-frequency resolution, which provides more possibility and higher accuracy for the detailed information extraction of complicated non-stationary signals.

Inspired by the theory of SST and the advantages of the GST, we propose a novel time-frequency analysis method that we havenamed synchrosqueezing generalized S-transform (SSGST).

Principles

GST

Gao et al. (2003) use a modulated harmonic wave with four undetermined parameters to replace the basic wavelet in ST to overcome the disadvantage caused by the fixed basic wavelet function. The modulated harmonic wave is written as,

$$w_f(t) = A|f|\exp\left[-\alpha\left(ft-\beta\right)^2 - i2\pi f_0 ft\right] \tag{1}$$

So the generalized S-transform (GST) is:

$$GST_x(f,b) = A|f|\int_{-\infty}^{\infty} x(t)\exp\left\{-\alpha\left[f(t-b)-\beta\right]^2\right\}\exp\left(-i2\pi f_0 ft\right)dt \tag{2}$$

Where, $x(t)$ represents a signal, GST_x denotes the generalized S-Transform of $x(t)$. f is frequency, t is time and b is the time shift. A is amplitude of the basic wavelet, α is energy attenuation ratio ($\alpha>0$), β is energy delay time and f_0 is video frequency of the basic wavelet.

SSGST

First, the Equation (2) can be reformulated as,

$$GST_x(f,b) = A|f|\exp\left(-i2\pi f_0 fb\right) \times\int_{-\infty}^{\infty} x(t)\exp\left\{-\alpha\left[f(t-b)-\beta\right]^2\right\}\exp\left[-i2\pi f_0 f(t-b)\right]dt \tag{3}$$

Let: $\psi(t) = A\exp\left[-\alpha(t-\beta)\right]\exp\left(i2\pi f_0 t\right)$

then Equation (3) is expressed as,

$$GST_x(f,b) = |f|\exp\left(-i2\pi f_0 fb\right)\int_{-\infty}^{\infty} x(t)\overline{\psi\left[f(t-b)\right]}dt \tag{4}$$

According to the Parseval theorem and transformation properties of scale and translation in Fourier Transform, the Equation (4) can be derived as follows:

$$GST_x(f,b) = \frac{1}{2\pi}\exp\left(-i2\pi f_0 fb\right)\int_{-\infty}^{\infty}\hat{x}(\omega)\overline{\hat{\psi}\left(f^{-1}\omega\right)}e^{ib\omega}d\omega \tag{5}$$

Then, we can calculate the instantaneous frequency of the signal $x(t)$ by using Equation (6):

$$f_x(f,b) = f_0 f + \left[i2\pi GST_x(f,b)\right]^{-1}\frac{\partial GST_x(f,b)}{\partial b} \tag{6}$$

$\overline{\psi(t)}$ is the complex conjugate of $\psi(t)$. $\overline{\hat{\psi}(\omega)}$ is the complex conjugate of Fourier Transform result of $\psi(t)$. $\hat{x}(\omega)$ denotes the Fourier Transform result of the signal $x(t)$.

SSGST is defined as the Equation (7) according to the theories of synchrosqueezing.

$$SSGST_x(f_l,b) = L_f^{-1}\sum_{f_k:\left|f_x(f_k,b)-f_l\right|\le\Delta f/2} GST_x(f_k,b)\exp\left(i2\pi f_0 f_k b\right)f_k^{-1}\Delta f_k \tag{7}$$

Where, f_l is the frequency of the SSGST result, b is the time. L_f is the half length of frequency range [f_l - L_f, f_l + L_f] centered on the frequency point f_l . f_k is the discrete frequency points in frequency ranges of the GST, and $\Delta f_k = f_k - f_{k-1}$。

The equation represents that the time-frequency spectra values among the frequency range [f_l - L_f, f_l + L_f] are superimposed on the frequency point f_l, so that the SSGST has higher accuracy of time-frequency decomposition ability.

The inverse transform of SSGST is given as,

$$x(b) = \mathrm{Re}\left[C_\psi^{-1}\sum_l SSGST_x(f_l,b)L_f\right] \tag{8}$$

Synthetic Examples

1. The Double Linear Chirped Signal

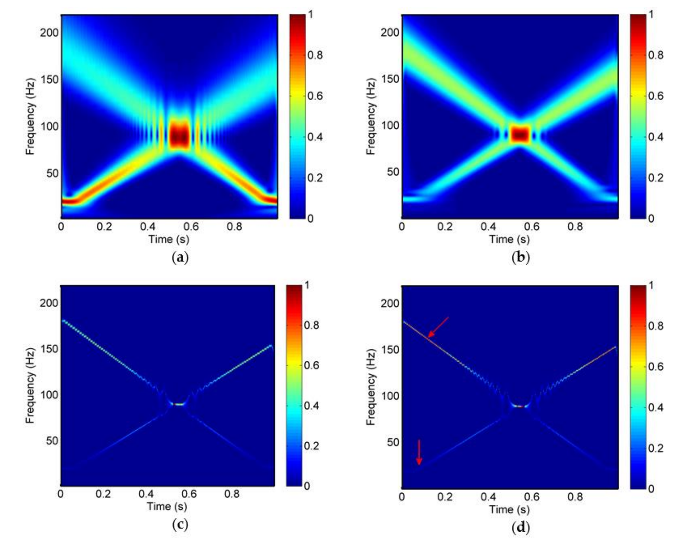

Figure 1. Time-frequency spectra of the double linear chirped signal based on (**a**) CWT, (**b**) GST, (**c**) SSCWT, and (**d**) SSGST.

2. The Double Hyperbolic Chirped Signal

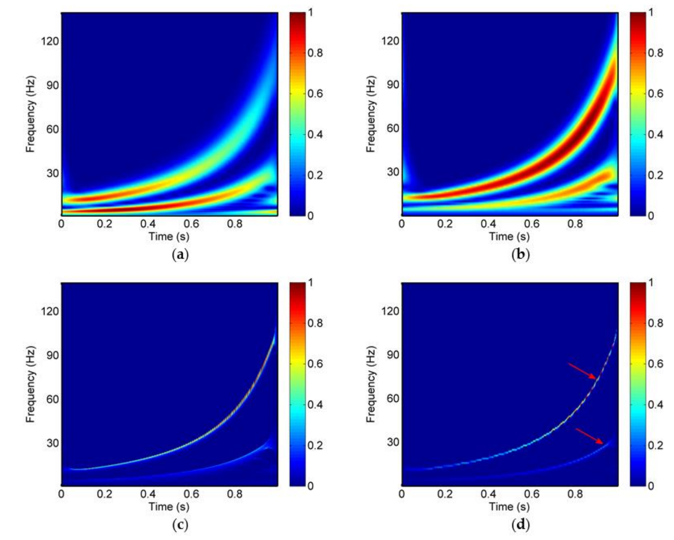

Figure 2. Time-frequency spectra of the double hyperbolic chirped signal based on (**a**) CWT, (**b**) GST, (**c**) SSCWT, and (**d**) SSGST.

3. A Synthetic Seismic Signal

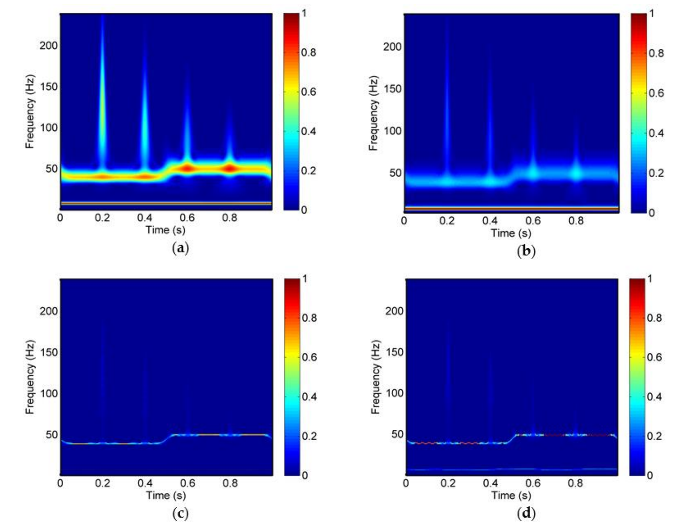

Figure 3. Time-frequency spectra of the synthesized signal based on (**a**) CWT; (**b**) GST; (**c**) SSCWT; (**d**) SSGST.

As can be seen, although all energy centers around the true instantaneous frequencies of the signal, the energy of the CWT and GST results smears heavily. In contrast, due to the effect of the "squeezing," SSCWT and SSGST squeeze all time-frequency coefficients into the time-frequency trajectory, which makes the spectra more energy-concentrated. In other words, SSCWT and SSGST show higher time-frequency resolution than CWT and GST. The arrows show that there is no distinct "steps" in the SSGST result compared with the result of SSCWT.

We adopted the mean square error (MSE) to validate the reconstruction ability of the SSGST method. As shown in Table, taking the MSE values of other time frequency transforms as references, the SSGST method can reconstruct the original signal well with a lower reconstruction error.

MSE	CWT	GST	SSCWT	SSGST
$Signal_1$	0.0121	3.6979×10^{-27}	2.3060×10^{-5}	1.8626×10^{-5}
$Signal_2$	0.0181	3.2963×10^{-27}	0.0214	0.0083
$Signal_3$	0.0221	5.0622×10^{-26}	7.5347×10^{-8}	1.0925×10^{-4}

Field Data Examples

When seismic waves propagate through hydrocarbon reservoirs, waves induced by fluid flow can lead to abnormal attenuation of energy and frequency. The phenomenon of abnormal attenuation mainly shows the loss of high-frequency energy and the conservation of strong low-frequency energy. Spectral decomposition technology can be utilized to identify hydrocarbon reservoirs by analyzing the different frequency response characteristics among different scale geological bodies. Here, we apply the SSGST method to seismic field data from the ZhongJiang Gas Field located in the western Sichuan Basin, China, in order to identify hydrocarbon reservoirs by analyzing the abnormal instantaneous frequency and energy.

In Fig4, the green curve in the horizontal direction at around 1.35ms is a seismic horizon and the green vertical line represents a gas well. The area within the blue ellipse is a gas-bearing reservoir, which is our study area. Fig5 is a histogram based on comprehensive analysis of log data. There are two sets of hydrocarbon reservoirs in the study area: the first set of reservoirs (JS33-1) is located in near a depth of 2560 m and thickness of about 22 m, and the second set of reservoirs (JS33-2) is located in near a depth of 2600 m and thickness of about 18 m.

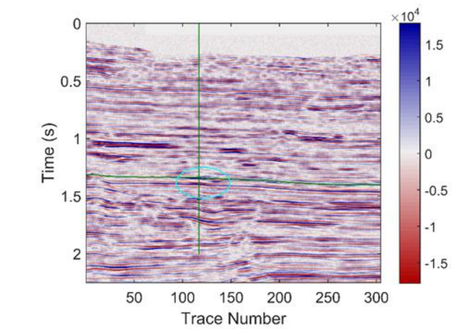

Figure 4. The seismic section from the ZhongJiang Gas Field.

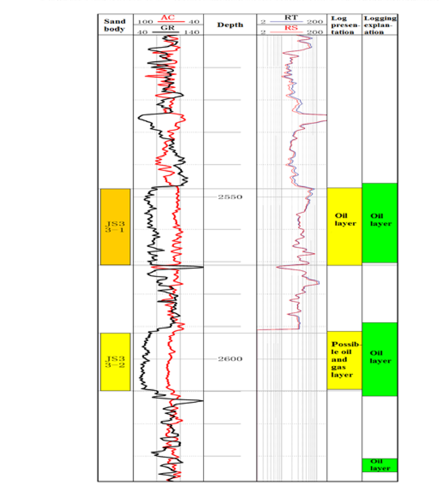

Figure 5. Histogram from the comprehensive analysis of well data

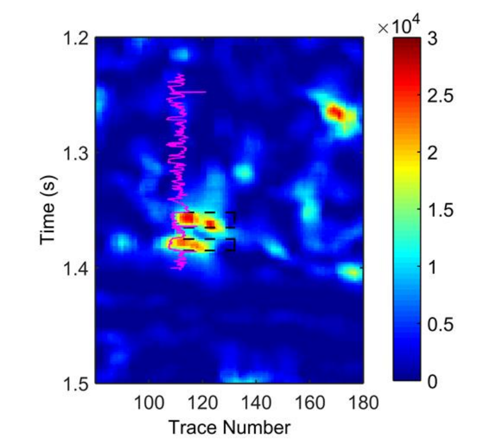

Figure 10. The details of Figure 9a near hydrocarbon reservoir.

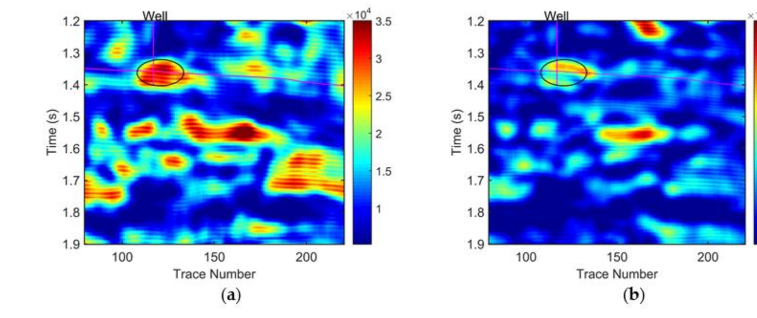

Figure 6. Constant frequency sections based on CWT (**a**) at 40 Hz and (**b**) at 50 Hz.

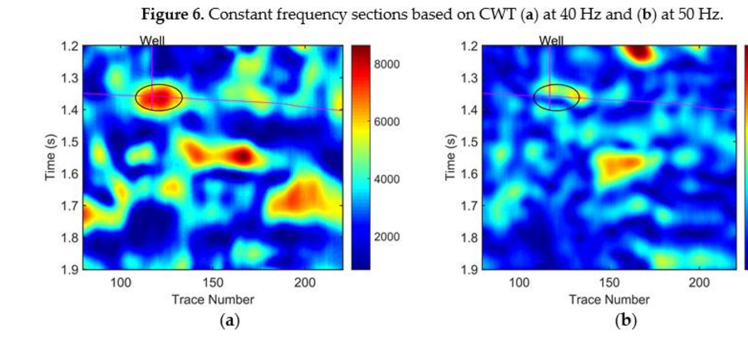

Figure 7. Constant frequency sections based on GST (**a**) at 40 Hz and (**b**) at 50 Hz.

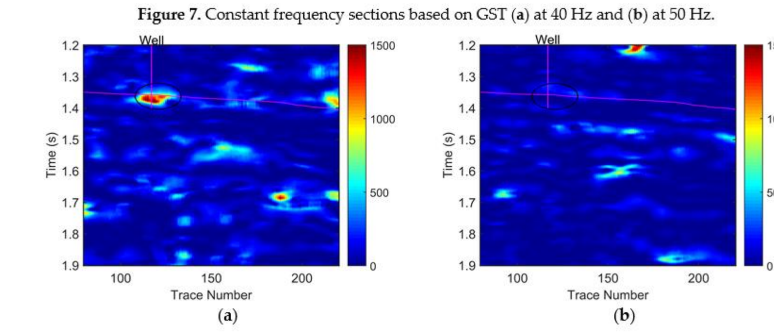

Figure 8. Constant frequency sections based on SSCWT (**a**) at 40 Hz and (**b**) at 50 Hz.

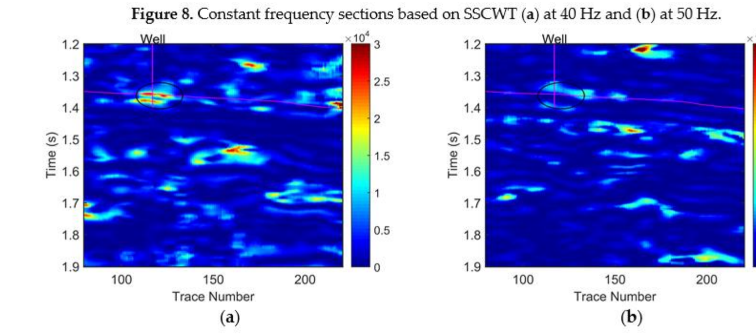

Figure 9. Constant frequency sections based on SSGST (**a**) at 40 Hz and (**b**) at 50 Hz.

Although the analysis results of the CWT and GST can all observe the abnormal attenuation in the hydrocarbon reservoir, they offer rough reservoir information due to the poor time-frequency resolution of CWT and GST. Compared with the CWT and GST methods, the SSCWT method can improve the time-frequency resolution and obtain the accurate reservoir information but cannot identify the boundaries of the two sets of hydrocarbon reservoirs, as shown in Figure 8.

we have locally enlarged the 40 Hz constant frequency section in Fig9 and combined it with the red acoustic velocity logging curve to analysis. The strong energy group in the two black-dashed, rectangular areas in Fig10 represent the two hydrocarbon reservoirs, namely JS33-1 and JS33-2, respectively. According to the comparison with CWT, GST, and SSCWT, we can find that SSGST can provide higher time-frequency resolution and can more accurately extract the abnormal response characteristics of seismic signals. Therefore, the SSGST method can locate hydrocarbon reservoirs effectively and depicts the reservoir boundary accurately with higher precision.

Acknowledgment

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