Volumetric Attributes: Constrained least-squares spectral analysis – Program \textit{spec\_clssa}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{spec_clssa.png}
\caption{\textit{spec\_clssa} computation flow chart}
\end{figure}

**CONstrained LEAST-SQUARES SPECTRAL ANALYSIS – PROGRAM spec\_clssa**

Alternative Spectral Decomposition Algorithms

Spectral decomposition methods can be divided into three classes: those that use quadratic forms, those that use linear forms and those that use atom decomposition. Quadratic forms are based on Wigner-Ville distribution, which can be easily designed but lose the phase component of the data and therefore cannot be used in reconstruction. Linear forms are based on short time Fourier transform (STFT), and include the S transform and the continuous wavelet transform (or CWT) as in program \textit{spec\_cwt}. Program \textit{spec\_clssa} is a nonlinear implementation of the short time Fourier transform. Atomic decomposition reconstructs the signal by using small “atom-sized” signals (in our case wavelets), such as complex matching pursuit (program \textit{spec\_cmp}) and the Hilbert-Huang transform.

While there is only one input file to program \textit{spec\_clssa}, there are many output files which can easily fill your disk drives. Detailed spectral analysis will typically be done about a reservoir or other zone of interest, such that the input data volume may be windowed. If this is not the case, it is best to first run several of the statistical summary volumes including the peak magnitude, peak frequency, and peak phase, and phase residue entire volume. These spectra may be sufficient for your analysis. If not, the peak frequency volume will serve as a guide as to which spectral components are well tuned.
Volumetric Attributes: Constrained least-squares spectral analysis – Program \texttt{spec\_clssa}

Figure 1. The flow chart for program \texttt{spec\_clssa} – constrained least-squares spectral analysis.

**Computing spectral components**

To begin, click the \textit{Volumetric Attributes} tab in the \texttt{aaspi\_util} window and select the program \texttt{spec\_clssa}:
Volumetric Attributes: Constrained least-squares spectral analysis – Program **spec_clssa**

Program **spec_clssa** performs spectral decomposition using a constrained least-squares spectral analysis method. The following window appears:
Volumetric Attributes: Constrained least-squares spectral analysis –
Program spec_cIssa

You will note that many of the attributes appear to be similar to those of program spec_cmp, spectral decomposition using a complex matching pursuit algorithm. First, enter the (1) the name of the Seismic Input (*.H) file you wish to decompose, as well as a Unique Project Name and Suffix as you have done for other AASPI programs.

Constrained least-squares spectral analysis (clssa)

Puryear et al. (2012) proposed an inversion-based algorithm for computing the time frequency analysis of reflection seismograms using constrained least-squares spectral analysis. The method resulted in spectra that have reduced window smearing for a given window length relative to the discrete Fourier transform irrespective of window shape, and a time-frequency analysis with a combination of time and frequency resolution that is superior to the short time Fourier transform and the continuous wavelet transform. As compared with the continuous wavelet transform, the method has improved temporal resolution, particularly at low frequencies.

Seismic spectral decomposition is a trace-by-trace operation. Because each 1D seismic trace is converted to a 2D time-frequency panel, the process expands the dimensions of the original data via a non-unique transformation, suggesting an inversion-based approach to the problem. Several investigators have used different empirical criteria to define inversion-based spectral analysis methods. Following the Portniaguine and Castagna (2004) approach to seismic wavelet decomposition and reflectivity inversion, we invert the normal equations by applying an iteratively reweighted least-squares regularization algorithm to the complex spectral decomposition inverse problem as described in the box “Mathematical Implementation”.
Mathematical Implementation

Start with the definition of the forward problem.

\[ \mathbf{Fm} = \mathbf{d}, \]  
(A1)

where \( \mathbf{F} \) is the kernel matrix composed of complex sinusoidal basis functions,

\[ F_{\alpha \mathbf{i}} = \exp\{i(2\pi \alpha \Delta fT)\} = \cos(2\pi \alpha \Delta fT) + i \sin(2\pi \alpha \Delta fT). \]

(A2)

\( \mathbf{m} = m_0(2\pi k\Delta f) \) is the model parameter vector (i.e., the desired spectral coefficients), and \( d_\alpha = d(\alpha \Delta T) \) is the windowed seismic amplitude data. Forming the normal equations by multiplying both sides of equation A1 by \( \mathbf{F}^H \), the complex conjugate or Hilbert transpose of \( \mathbf{F} \), gives

\[ \mathbf{F}^H \mathbf{F} \mathbf{m} = \mathbf{F}^H \mathbf{d}. \]

(A3)

The objective of the inverse problem is to compute \( \mathbf{m} \) given \( \mathbf{F} \) and \( \mathbf{d} \). The classical least-squares solution to equation A1 is then

\[ \mathbf{m} = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{d}. \]  
(A4)

Conventional Fourier transforms used in filtering are constructed to be orthogonal, such that \( \mathbf{F}^H \mathbf{F} = \mathbf{I} \), where \( \mathbf{I} \) is the identity matrix. In this case, equation A4 reduces to

\[ \mathbf{m} = \mathbf{F}^H \mathbf{d}. \]  
(A5)

which is equivalent to the discrete Fourier transform of the windowed trace segment. If the analysis window is \( T = 0.1 \) s, orthogonality requires \( \Delta f = 1/T = 10 \) Hz. If we wish to estimate intermediate spectral components at a 2 Hz interval, we need to use the more general equation A4. A finer frequency increment results in an underdetermined system with more unknowns than equations. We constrain the inversion by introducing two diagonal matrices. The data weight matrix, \( \mathbf{W}_d \), acts as a taper, where smaller weights at the window edges provide for greater tolerance of the misfit. In contrast, the model weight matrix, \( \mathbf{W}_m \), change iteratively, in general favoring the strongest spectral components, and attenuating the corresponding weaker aliases. Applying \( \mathbf{W}_m \) and \( \mathbf{W}_d \) to equation A1,

\[ \mathbf{W}_m \mathbf{F} \mathbf{W}_d (\mathbf{W}_m)^{-1} \mathbf{m} = \mathbf{W}_d \mathbf{d}. \]  
(A6)

we introduce the weighted quantities

\[ \mathbf{F}_w = \mathbf{W}_m \mathbf{F} \mathbf{W}_d \quad \text{and} \quad \mathbf{m}_w = (\mathbf{W}_m)^{-1} \mathbf{m}. \]  
(A6)

and then recast equation A5 as a model and data-weighted ill-posed inverse problem:

\[ \mathbf{F}_w \mathbf{m}_w = \mathbf{W}_d \mathbf{d}. \]  
(A7)

To solve equation A7, we add regularization:

\[ \min \alpha^2 = ||\mathbf{F}_w \mathbf{m}_w - \mathbf{W}_d \mathbf{d}||^2 + \alpha ||\mathbf{m}_w||^2. \]  
(A8)

where \( \alpha \) is a regularization parameter that can be varied to control the sparsity and stability of the solution. Note that if \( \mathbf{W}_m \) is proportional to \( \mathbf{m} \), small values of \( \mathbf{m} \) become large values of \( \mathbf{m}_w \), which are then minimized, giving us a sparse solution. Using an analytical Lagrange solution to equation A8, we solve first for the weighted model parameters

\[ \mathbf{m}_w = \mathbf{F}_w^H (\mathbf{F}_w \mathbf{F}_w^H + \alpha \mathbf{I})^{-1} \mathbf{W}_d \mathbf{d}. \]  
(A9)

followed by reconstruction

\[ \mathbf{m} = \mathbf{W}_m \mathbf{m}_w. \]  
(A10)

Values of \( \alpha \) are chosen to be a percentage of \( ||\mathbf{F}_w \mathbf{F}_w^H|| \). Portniaguine and Castagna (2004) find the CLSSA utilizing a real basis function to be not quite as good as using a complex basis function, but better than the conventional DFT. This is significant because CLSSA using only the real waveform has precisely the same temporal resolution as the Fourier transform while having greatly improved frequency resolution. This indicates that, for a given window, CLSSA has a better Heisenberg uncertainty product (standard deviation of the waveform in time multiplied by the standard deviation of the spectrum) than does the Fourier transform. Thus far, they have not found a window for which this is not the case.
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

The Reconstruction parameters (2) indicates the four corner frequencies $f_1$, $f_2$, $f_3$, and $f_4$ used to reconstruct the seismic data after flattening. A raised-cosine taper is applied between $f_1$ and $f_2$ as well as between $f_3$ and $f_4$. The choice of frequency increment and range does not significantly impact the decomposition time, only the output size of the data if you choose to output either spectral magnitude or spectral phase components as either a sequence of constant frequency volumes or as an equivalent 4D volume. In this example we used a 40 ms window, which implies a frequency resolution of $\Delta f = 1 / 0.040 \text{s} = 25 \text{Hz}$ is necessary to reconstruct the data using a DFT. Using this argument, we can say that the spectra is oversampled, which is validated by the excellent reconstruction on the right.

The (3) Window length indicates the length of the time window function used in spectral decomposition. As discussed above, the CLSSA algorithm is focused on solving an inverse problem. The window length parameter determines how long the inverse operator would cover, which can affect the time-frequency resolution. As an iterative method, the (4) Number of iterations defines the number of iterations used in iteratively reweighted least-squares solution. A larger number of iterations provides a more accurate least-squares approximation, but also consumes more time. The (5) alpha scaling parameter is the regularization parameter $\alpha$ in equations A8 and A9 above, which stabilizes the least-squares regression process.

The (6) Phase residue threshold (Percent) limits the phase residue attribute computation. When it is larger, more phase residue will appear corresponding to weaker events.

Comment [M1]: Marcilio – Is this true? What does the threshold do?
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

The Phase Residue

Ghiglia and Pritt (1998) provide an excellent survey of 2D phase-unwrapping techniques and show how a complex residue theorem based on vector calculus can be applied to the phase-unwrapping problem. They use a rectangular integration path aligned with the \( t \) and \( f \) axes. We choose a smaller diamond-shaped integration path about each sample \((j\Delta t, k\Delta f)\) given by

\[
I_{jk} = \frac{W[\psi(t_{j+1}, f_k) - \psi(t_j, f_k)]}{2\pi} + \frac{W[\psi(t_{j}, f_{k+1}) - \psi(t_{j+1}, f_k)]}{2\pi}
\]

where \( \psi \) is the phase and \( W \) is a wrapping operator that produces an output that falls between \( \pm \pi \). If the integral \( I_{jk} \) in equation B1 is nonzero, there are inconsistent phase points, which Ghiglia and Pritt (1998) call “phase residues”. The figure below shows how the residue is calculated for a small portion of a typical wrapped time-frequency phase matrix. Area A is continuous, with a phase residue = 0.0, while area B is discontinuous with a phase residue = 1.0.

Figure B1. Line integrals about analysis points A and B used to compute the phase residue.

Matos et al. (2011) find phase attributes are sensitive to the same kinds of stratigraphic discontinuities seen by analyzing the magnitude component of time-frequency distribution using wavelet transforms and the continuous wavelet transform. Because phase is often a more seismic measure than magnitude, it holds significant promise in mapping stratigraphic unconformities.

Accurate computation of phase residues requires relatively fine sampling across frequency. We suggest setting the computational \( \Delta f = 1 \) Hz. Unfortunately, if you do this and choose to output these many components as well, you may fill up your disk drive. To analyze individual components we suggest a coarser output \( \Delta f = 5 \) Hz for subsequent loading into your interpretation workstation.
Volumetric Attributes: Constrained least-squares spectral analysis – Program *spec_clssa*

**Statistical measures of the spectrum**

Like program *spec_cmp*, program *spec_clssa* provides several statistical measures of the spectrum that can be used in addition to or in place of the full 4D spectral components. The peak spectral magnitude, peak spectral frequency, and peak spectral phase are easy to understand. You obtain these by placing a checkmark in front of *Want peak attributes?*
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

**Statistical measures of the spectrum**

Gaussian statistics such as the mean, standard deviation, kurtosis, and skewness are sometimes used to represent a seismic spectrum, with the mean representing the average spectrum, the standard deviation the bandwidth, and kurtosis and skewness deviations from the Gaussian spectrum model. Unfortunately, seismic processors try as hard as they can to make the spectra flat, which is decidedly non-Gaussian. Zhang (2010) therefore constructed a suite of attributes that better define these kinds of spectra. The local bandwidth is defined as the difference between user-defined percentiles. The range-trimmed mean is simply the average frequency within these percentiles. The slope is a measure of how the spectrum changes with frequency – e.g. increasing, flat, or decreasing. Finally, the roughness is a measure of local smoothness of the spectrum.

![Graph showing statistical measures of a spectrum](image.png)

Figure B1. The shape of a spectrum (in green) containing 50 or more components approximated by average frequency, bandwidth, slope, and roughness attributes.

**Individual spectral components**

If you place a checkmark in front of *Want spec mag cmpt?* or *Want spec phase cmpt?*, you will obtain each of the spectral components that range between $f_1$ and $f_4$ with the desired *Frequency Increment*. For interpretation of the components on most interpretation workstations, it may be easier to load these components separately. If you
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

place a check mark in front of (9) Store cmpts as 4D cubes?, you obtain spectral gatherings that are ordered with the time axis running fastest, followed by the frequency axis, (such that the first two indices represent a time-frequency distribution) followed by the CDP numbers (inline axis) and line numbers (crossline axis). The 4D volumes will have the following names for this job:

If you ask for spectral components not to be stored as a 4D cube the constant-frequency 3D spectral magnitude and spectral phase volumes will have the frequency value encoded in the file name:

There are several ‘expert’ controls under the Extended tab:
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

Since the amount of output can be quite large, it may be useful to run `spec_clssa` on only a limited range of (1) inlines and (2) crosslines.

You will probably want to experiment with these parameters a bit to calibrate them for the kind of data you encounter. It is reasonable to expect that most surveys of a similar vintage from the Gulf of Mexico will have similar spectral ranges and signal-to-noise ratios. Likewise, similar surveys acquired in the Permian Basin of west Texas will be similar to each other. In order to simplify parameter choices, you can then `cat AASPI "parms"` file to examine its content or set it to your desired default parameters:
Volumetric Attributes: Constrained least-squares spectral analysis – Program **spec_clsssa**

The file in your working directory will always take precedence over the one in the `$(AASPIHOME)/scripts` directory.

As in all the AASPI GUIs, click **Execute** to run the program. The end of your run should looks something like the following:
Example 1: Comparison of the three decomposition algorithms

Our first example compares vertical slices of spectral components at 20 Hz and 50 Hz through the UT BEG Stratton data volume. Note that 20 Hz has a period of 0.050 s which is larger than the 0.040 s analysis window used in `spec_clssa`. The 20 Hz component from `spec_cwt` is not really a component, but rather the relatively broad band response of a Morlet wavelet centered at 20 Hz, with a Gaussian spectrum with half power $f = f$. The 20 Hz component from `spec_cmp` is the complex sum of all 20 Hz components of Morlet wavelets about a given time sample. For this reason, both `spec_clssa` and `spec_cmp` have higher frequency resolution than `spec_cwt`.

Comment [M2]: Marcilio – what is the bandwidth (i.e. standard deviation) of a 20 Hz Morlet wavelet?
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clsssa
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

Example 2: clssa applied to the Boonsville survey

Now, plot some of the results. Since we did not choose to store the spectral magnitude and phase components as a 4D cubes, we have several 3D volumes we can plot separately.

Plotting the same time slice as in all the other examples, and setting Allpos=y in our SEP Viewer GUI for the strictly positive magnitude, the `spec_mag_3d_clssa_d_20.H` (the 20 Hz magnitude component) file looks like this:
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

While the `spec_mag_3d_clssa_d_50.H` (the 50 Hz magnitude component) file looks like this:

The phase components will range from $-180^0$ to $+180^0$, so set Allpos=n and choose a cyclical color bar to plot `spec_phase_3d_clssa_d_20.H` (the 20 Hz phase component):
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

and `spec_phase_3d_clssa_d_50.H` (the 50 Hz phase component):
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

Several measures of the spectrum at each time sample are also computed. The simplest one is the peak magnitude (the greatest value of the spectrum) here as the file `peak_magnitude_clssa_d.H`:

We can also plot the corresponding frequency at this value (the peak frequency) which for this case is the file `peak_freq_3d_clssa_d.H` plotted against the `frequency.sep` (magenta-red-yellow-green_cyan_blue) color bar:
Plotting Spectral Components

We provide a simple graphical interface to quality control the spectral components. Many commercial workstation software products now provide excellent interactive visualization of 4D volumes ($t$, $x$, $y$, and typically offset $h$, but in our case frequency, $f$). Our crude tool `plot_4D_spectral_components` can be found under the Display tools tab:
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

Previously, I had computed spectral components for the Boonsville survey and stored them as a 4D volume \((t,f,\text{line no},\text{cdp no})\) in a file \textit{spec\_phase\_4d\_clssa\_d\_0.H}\n
\[\text{spec\_phase\_4d\_clssa\_d\_0.H}\]
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

The upper selection bar allows me to plot a constant frequency section or a constant time section. I've chosen the 1.1s time slice. I choose the energy.sep color bar and obtain these slices of different frequencies at 1.1s time slice:
Volumetric Attributes: Constrained least-squares spectral analysis – Program **spec_clssa**

First, 5 Hz component at 1.1s:

![spectral phase components](image)

Then, 35 Hz component at 1.1s:
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa
The upper selection bar allows me to plot a constant frequency section or a constant time section. I've chosen the 20 Hz component. I choose the energy.sep color bar and obtain these time slice through a 20 Hz volume:
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clssa`

First, 0.8s time slice of 20 Hz component:

![Image of spectral phase components frequency](image)

Then, 1.4s time slice of 20 Hz component:
Volumetric Attributes: Constrained least-squares spectral analysis – Program *spec_clssa*

I can also generate similar plots of the *spec_mag_4d_clssa_0.H* 4D volume.
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

I display the 31 Hz and 41 Hz spectral magnitude slices at 1.1 s:

First, 31 Hz spectral magnitude slices at 1.1 s:
Volumetric Attributes: Constrained least-squares spectral analysis – Program `spec_clusa`

Then, 41 Hz spectral magnitude slices at 1.1 s:
Volumetric Attributes: Constrained least-squares spectral analysis – Program spec_clssa

References